Journal of Money, Investment and Banking
ISSN 1450-288X Issue 17 (2010)

http://www.eurojournals.com/JMIB.htm

Do the Recession Durations in the Economy follow Heavy-Tailed Distributions? The case of the USA 1791-2008

Paraschos Maniatis
Department of Business Administration at Athens University of Economics and Business
76 Patission St., Athens, GR-104 34
E-mail: pman@sch.gr
&
Kuwait-Maastricht Business School, MBA Program, Dasma-Kazima Street
P.O. Box: 9678 Salmiya, 22097 Kuwait
E-mail: paris@kmbs.edu.kw

Abstract

In the quantitative economics and in the econometrics the largest portion of probability distributions is held by the normal distribution, mainly because of its invaluable analytical power and the easiness of its use. However, phenomena, usually qualified as 'rare', such as panics, crises, and recessions' duration cannot be satisfactorily treated by the normal distribution- even worst it misleads the user giving certainty, where it doesn't exist. The financial crises of the 90s and the last global recession after the financial crisis of 2008, destroyed the blind belief to the normal distribution and gave room to distributions, which better handle extreme phenomena in the economy, namely the heavy-tailed, unilateral or bilateral, distributions and especially to extreme values distributions. In the present study, we apply several of these distributions in an effort to estimate the duration of the most recent recession sparked by the financial crisis of October 2008. As source of data we use the list of recessions in the USA during the period 1791-2008. The analysis shows that the Fréchet distribution and the Burr distribution give the best estimates for the recession duration. Further, the study shows that the difference between expansions/duration in the same cycle follows a Cauchy distribution and that the ratio recession duration to expansion duration follows a Weibull distribution with diminishing shape parameter.

Keywords: Heavy-tailed distributions, Extreme values distributions.

Introduction

The heavy-tailed distributions and the extreme values distributions were first employed to describe and forecast natural catastrophic phenomena as floods, earthquakes, hurricanes, etc. Fréchet (1927), Zipf (1947) and Gumbel (1958) discovered the homonymous probability distributions, on which is based the modern theory of extreme values in statistcs. Since the 60's, mainly inspired by the work of Mandelbrot (1960, 1997) efforts were made to apply the extreme phenomena distributions to economic variables, in the first place in the stock exchange variations and the risk management in the financial products. The daily fluctuations of the Dow Jones index of the industrial stocks values in the New York stock exchange gives an impressive idea illustration of the enhanced number of extreme variations.

Since 1897 the New York stock exchange was open about 29,000 days without dramatic variations with the exception of the crises of the years 1929, 1987 and 2008. In the following table 1 are shown the number of extreme variations of the index, from zero measured in terms of standard deviation.

Table 1: Extreme variations of the Dow Jones industrial stocks index in the New York Stock exchange

Size of variation	Observed number of times the index exceeded the size	Theoretical number according to the normal distribution
6,83	20	$6x10^{-8}$
-6,91	20	$3x10^{-8}$
11,08	5	$1x10^{-24}$
-11,73	3	$6,5x10^{-28}$
15,34	1	3x10-49
-22,61	1	$2,5x10^{-109}$

Source: Zajdenweber: Economie des extremes

The above example is a good example of misleading results that can be obtained by the imprudent application of the normal distribution in the economic life. The examples can be multiplied when we refer to the last financial crisis of 2008, where all the risk appraising models based on the normal distribution broke down. A financial crisis is always followed by a recession in the real economy, which cal last from a few months to several years, depending on the strength of the crisis and the measures taken by the authorities. Starting from several examples of extreme variation in the economy the French mathematician Mandelbrot (1997, 2004) strongly disputed the use of the normal distribution, speaking for the "Gaussian dogma" in the economic and financial fields. Mandelbrot proposed other types of probabilistic distributions with or without mean and/or variance, which can, nevertheless, deal with phenomena, which the "Gaussian dogma" cannot. These distributions are labeled as "heavy-tailed", in the sense that they allow for positive probabilities (although small) for the stochastic variable to exceed extreme values. Formally speaking a probability distribution is characterized if it is not exponentially bounded i.e. they have tails heavier than the exponential distribution. Since the extreme value distributions fall in the family of the heavy-tailed distributions they are compared to the other heavy-tailed distributions in the study of rare but not improbable phenomena.

Placing the recessions in the frame of the entire economic cycle we posed the following questions:

- Does a heavy-tailed distribution give plausible estimates for the duration of recent recession?
- How is distributed the difference expansion-recession duration?
- How is distributed the ratio recession to expansion duration?

The Data and the Variables

The data consists of the duration of the expansions, the recessions and the cycles in the USA economy and cover the period 1791-2008. The choice of this series is due to that the USA data in this field is the most accurate and complete and available to all interested people. The data is shown in the appendix II to this text and cover the start and the end of the expansions and the recessions. The calculation of the expansions, recessions and cycles duration is made by us.

We used the following abbreviations to name the variables

EXD: expansion duration in a cycle
RCD: recession duration in the same cycle
CYD: cycle duration measured from through to through=EXD+RCD
EXD-RCD: duration of expansion minus duration of recession in the same cycle
RCD/EXD: ratio of the recession duration to the expansion duration in the same cycle

All durations are measured in month.

Descriptives of the Variables

As shown in the table 22 during the period 1791-2008, that is in 217 years, 47 panics and crises interrupted the smooth growth of the American economy, giving an average of 2,16 appearances of such phenomena every 10 years. The average duration of each cycle was 55,62 month, the expansion average duration was 36,31 month, while the average recession duration was 19,3 month. Besides, the distributions of the expansion durations and the recession durations exhibit positive skewnness, with the recessions' skewness greater than this of the expansions. The details of the recessions, the expansions and the cycle durations are shown in the tables 2 through 4 below.

Table 2: Descriptives of the variable EXD

EXD- Statistic	Value	Percentile	Value
Sample Size	47	Min	10
Range	110	5%	12
Mean	36,3191	10%	12
Variance	623,744	25% (Q1)	20
Std. Deviation	24,9749	50% (Median)	27
Coef. of Variation	0,68765	75% (Q3)	45
Std. Error	3,64296	90%	74,4
Skewness	1,6804	95%	100,4
Excess Kurtosis	2,7214	Max	120

Table 3: Descriptives of the variable RCD

RCD- Statistic	Value	Percentile	Value
Sample Size	47	Min	6
Range	66	5%	6,4
Mean	19,2979	10%	8
Variance	207,605	25% (Q1)	11
Std. Deviation	14,4085	50% (Median)	13
Coef. of Variation	0,746637	75% (Q3)	23
Std. Error	2,1017	90%	39,0
Skewness	2,0937	95%	58,2
Excess Kurtosis	4,61175	Max	72

Table 4: Descriptives of the variable CYD

CYD- Statistic	Value	Percentile	Value
Sample Size	47	Min	24
Range	104	5%	24
Mean	55,617	10%	28
Variance	723,546	25% (Q1)	36
Std. Deviation	26,8988	50% (Median)	48
Coef. of Variation	0,483643	75% (Q3)	72
Std. Error	3,92359	90%	101,6
Skewness	1,06626	95%	113,4
Excess Kurtosis	0,308706	Max	128

Table 5: Descriptives of the variable EXD-RCD

EXD-RCD Statistic	Value	Percentile	Value
Sample Size	47	Min	-36
Range	148	5%	-34,0
Mean	17,0213	10%	-13,2
Variance	939,152	25% (Q1)	0
Std. Deviation	30,6456	50% (Median)	12
Coef. of Variation	1,80043	75% (Q3)	35
Std. Error	4,47012	90%	57,6
Skewness	1,08924	95%	90,6
Excess Kurtosis	1,80159	Max	112

Table 6: Descriptives of the variable RCD/EXD

RCD/EXD Statistic	Value	Percentile	Value
Sample Size	47	Min	0,06667
Range	3,93333	5%	0,092176
Mean	0,776814	10%	0,103706
Variance	0,527376	25% (Q1)	0,26027
Std. Deviation	0,726207	50% (Median)	0,5
Coef. of Variation	0,934852	75% (Q3)	1
Std. Error	0,105928	90%	1,91274
Skewness	2,27827	95%	2,02857
Excess Kurtosis	7,42637	Max	4

Modeling the Recessions Data

Several types of models can be employed to describe phenomena with heavy-tailed distributions in economics. The most popular heavy-tailed models include extreme values distributions such as the Gumbel max, the Weibull and the Fréchet distributions, which are special cases of the Generalized Extreme Value (GEV) model and they can be obtained by proper manipulation of the GEV parameters and further other types of heavy-tailed models.

The cumulative probability functions (cdf) of the models we considered are as follows:

Gumbel max distribution $F(x)=\exp(-(\exp(-z)))$ $z=(x-\mu)/\sigma$ (1)

$-\infty < x < \infty$

σ: scale parameter ($\sigma>0$), μ: location parameter

Weibull distribution $F(x)=1-\exp[-((x-\gamma)/\beta)^{\alpha}]$ (2)

$\gamma < x < \infty$

α: shape parameter ($\alpha>0$), β: scale parameter ($\beta>0$), γ: location parameter

Fréchet distribution $F(x)= \exp[-(\beta/(x-\gamma))^{\alpha}]$ (3)

$\gamma < x < \infty$

α: shape parameter ($\alpha>0$), β: scale parameter ($\beta>0$), γ: location parameter

Generalized Extreme Value distribution (GEV)

for $k\neq0$ $F(x)=(1/\sigma)\exp(-(1+kz)^{-1/k})(1+kz)^{-1-(1/k)}$ $z=(x-\mu)/\sigma$ (4)

in the domain $1+k[(x-\mu)/\sigma)] > 0$

for $k=0$ $F(x)=(1/\sigma)\exp[-z-\exp(-z)]$ $z=(x-\mu)/\sigma$ (4.1)

$-\infty < x < \infty$

k: shape continuous parameter, σ: scale parameter ($\sigma>0$), μ: location parameter

The form of the above distributions imposes a lower limit to the Weibull and Fréchet distribution, while the domain of GEV distribution in case that $k\neq0$ depends on the sign of k.

Other heavy-tailed distributions used in modeling economic variables are:

One-tailed distributions:

Weibull distribution with shape parameter <1

Lognormal distribution $F(x)=\Phi[(\ln(x-\gamma)-\mu)/\sigma]$ Φ: Laplace integral (5)

$\gamma < x < \infty$

σ: scale parameter (σ>0), μ: location parameter, γ: location parameter

- **Burr** distribution $F(x)=1-[1+((x-\gamma)/\beta)^{\alpha}]^{-k}$ (6)

$\gamma \leq x < \infty$

k: shape continuous parameter, α: shape parameter (α>0), β: scale parameter (β>0), γ: location parameter

- **Pareto** distribution of the 1st kind $F(x)=1-(\beta/x)^{\alpha}$ (7)

$\beta \leq x < \infty$

- **Pareto** distribution of the 2nd kind $F(x)=1-[\beta/(x+\beta)]^{\alpha}$ (7.1)

$0 \leq x < \infty$

α : shape parameter (α>0), β: scale parameter (β>0)

- **Lévy** distribution $F(x) = 2-2\Phi[(\sqrt{(\sigma/(x-\gamma))})]$ Φ: Laplace integral (8)

$\gamma < x < \infty$

σ: scale parameter (σ>0), γ: location parameter

Two-tailed distributions

Gumbel max distribution

Generalized Extreme Value distribution with shape continuous parameter k=0

Cauchy distribution $F(x)=(1/\pi)\arctan[(x-\mu)/\sigma] +0,5$ (9)

$-\infty < x < \infty$

σ: scale parameter (σ>0), μ: location parameter

- **Normal** distribution $F(x)=\Phi[(x-\mu)/\sigma]$ Φ: Laplace integral (10)

$-\infty < x < \infty$

σ: scale parameter (σ>0), μ: location parameter

All of the above distributions are heavy-tailed distributions (except Normal), while the Lévy, the Cauchy and the Normal distributions are stable distributions.

We fitted in the variables under investigation 61 different probability distributions focusing the interest in the degree in which the best fitting distributions are heavy-tailed distributions.

The Fitting Results

In the following table 7 are shown the rank of the best fitting distribution out of 61 distributions in the histograms of the variables and the rejection at level of significance 5% of the hypothesis that the variables follow the indicated distribution under the goodness of fit criteria of

- Kolmogorov-Smirnov (K-S)
- Anderson-Darling (A-D) and
- Chi-2

Table 7: Goodness of fit of heavy-tailed distribution in the histograms of variables

Table Variable	Distribution	K-S reject at 5 %	Rank K-S	A-D reject at 5%	Rank A-D	Chi-2 reject at 5%	Rank Chi-2
RCD	Gumbel max	**No**	30	**No**	24	**Yes**	36
RCD	Weibull	**No**	20	**Yes**	27	**No**	28
RCD	Frechet	**No**	2	**No**	1	**No**	7
RCD	Gen.Extr.Value	**No**	11	**No**	9	**No**	10
RCD	Lognormal	**No**	7	**No**	8	**No**	4
RCD	Burr	**No**	1	**No**	6	**No**	3
RCD	Pareto	**Yes**	40	**Yes**	46	**Yes**	32
RCD	Cauchy	**Yes**	32	**Yes**	32	**No**	27
RCD	Levy	**Yes**	37	**Yes**	40	**Yes**	42
RCD	Normal	**Yes**	39	**Yes**	36	**No**	12
EXD-RCD	Cauchy	**No**	1	**No**	12	**No**	3
RCD/EXD	Weibull	**No**	6	**No**	19	**No**	22

In the above table we read that for the variable RCD the goodness of fit of the Gumbel max, Weibull, Pareto, Cauchy, Lévy and Normal distributions is rejected by at least one of the posed criteria, so we discard them as candidates of good fitting distributions. Further looking at the rank of goodness of fit for the remaining distributions we see that the Fréchet and the Burr distributions have excellent ranks with small variations in the criteria.

For the variable EXD-RCD the Cauchy distribution is ranked as first by the K-S criterion and as third by the Chi-2, while a low rank is given by the A-D criterion.

For the variable RCD/EXD although the Weibull distribution is not rejected by any criterion, all three criteria give a low rank to this distribution

In the following table 8 are shown the values of the scale, location and shape parameters of the candidate distributions.

Table 8: Values of the scale, location and shape parameters of the candidate distributions

Variable	Distribut.	σ	μ	k	α	β	γ
RCD	Gumbel max	11,2343	12,8133				
RCD	Weibull				0,836771	14,0792	6,000
RCD	Frechet				2,15224	12,5866	-0,5132
RCD	Gen.Extr. Value	6,22587	12,2519	0,36386			
RCD	Lognormal	0,89585	2,2899				4,7098
RCD	Burr			0,34993	5,18627	9,9694	
RCD	Pareto				1,0297	6,0000	
RCD	Cauchy	4,28864	12,9779				
RCD	Levy	5,24397					5,2571
RCD	Normal	14,4085	19,2979				
EXD-RCD	Cauchy	12,2709	9,5928				
RCD/EXD	Weibull				0,93598	0,685558	0,0667

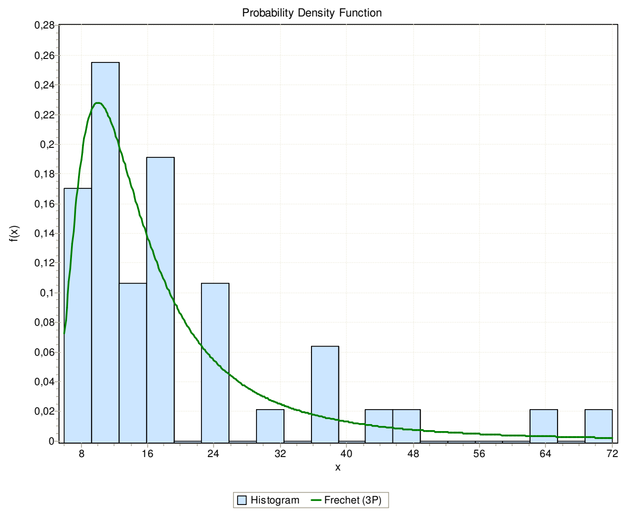
Presentation of the Graphs and the Goodness of Fit of the Adopted Distributions

The goodness of fit and the rank evaluation of the candidate distributions advocate to adopt the Fréchet and the Burr distributions as best distributions of the variable RCD the, Cauchy distribution as best candidate for the variable EXD-RCD and the Weibull distribution for the variable RCD/EXD.

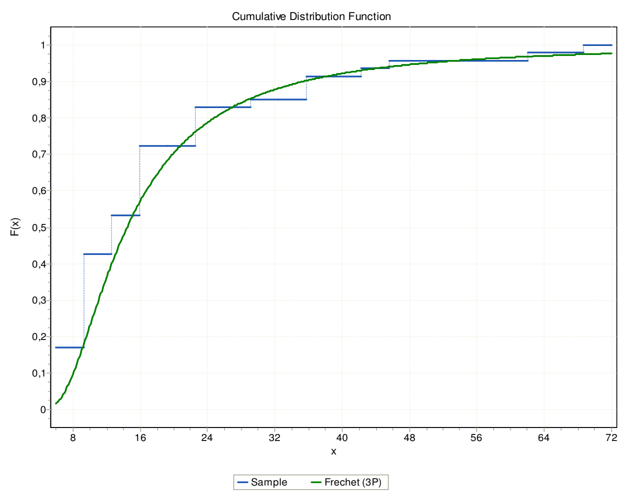
A look in the corresponding pdf and cpf graphs of these distributions support our decision. The details of goodness of fit for these distributions are shown in the tables 10, 11,12 and 13 accordingly.

The rest of the graphs and the goodness of fit tables for the discarded distributions are shown in the appendix I.

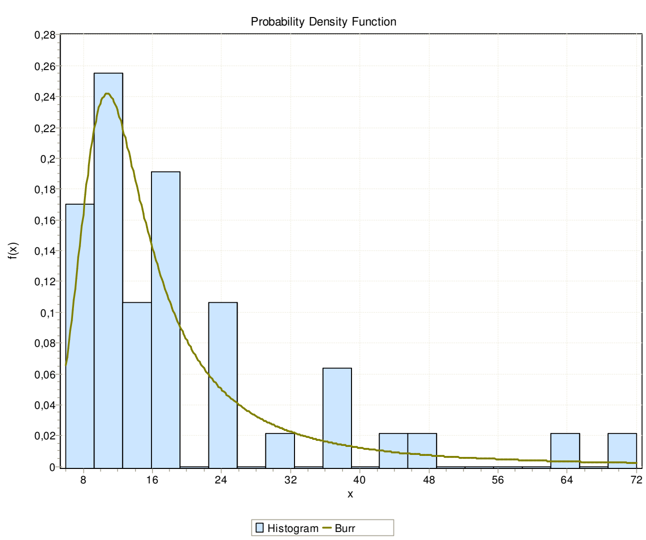
Graph 1: pdf of RCD- Fréchet



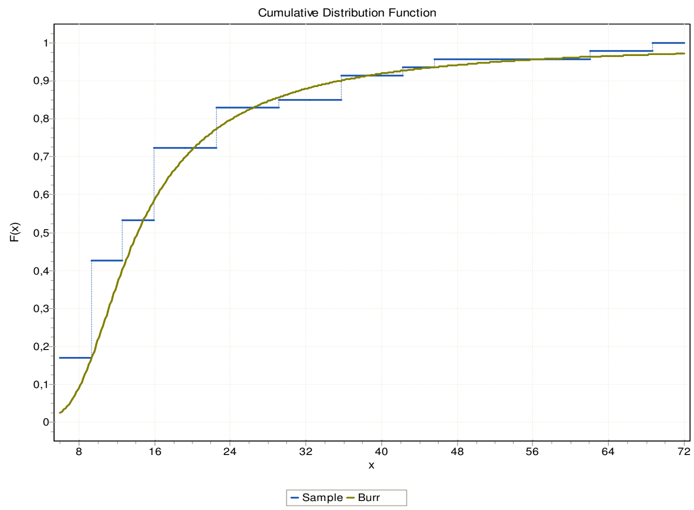
Graph 2: cdf of RCD- Fréchet



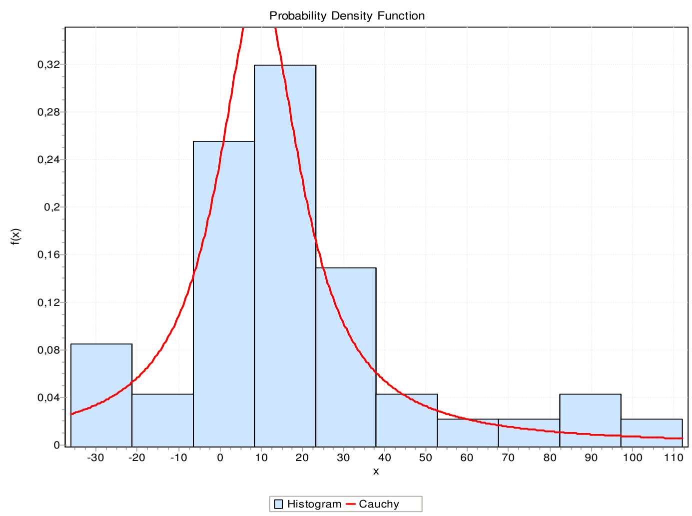
Graph 3: pdf of RCD- Burr



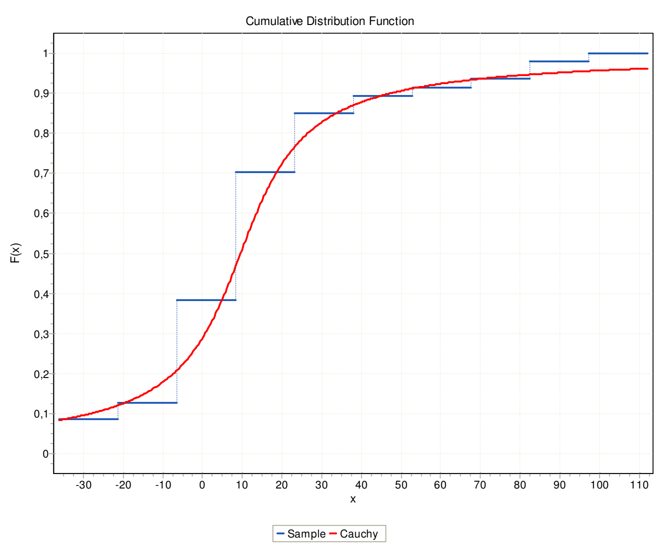
Graph 4: cdf of RCD- Burr



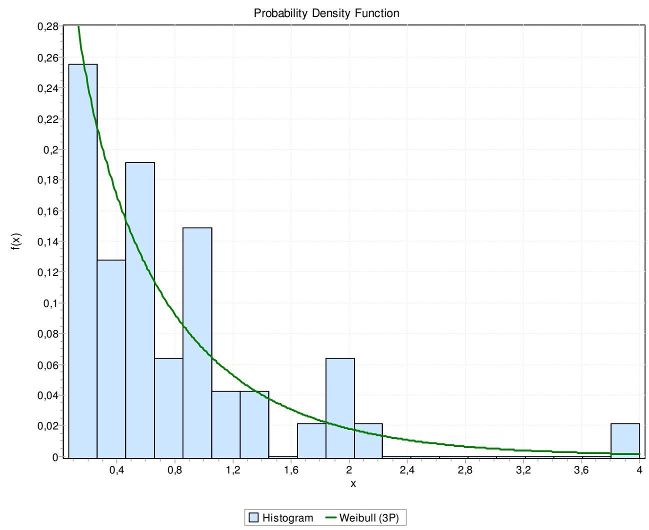
Graph 5: pdf of EXD-RCD Cauchy



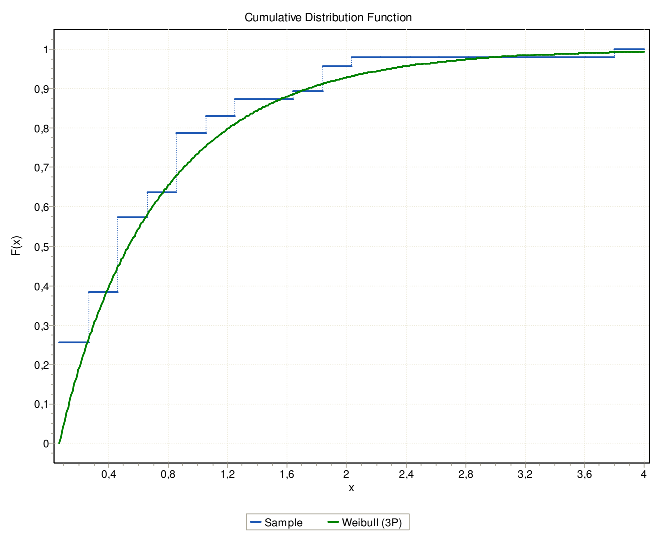
Graph 6: cdf of EXD-RCD Cauchy



Graph 7: pdf of RCD/EXD Weibull



Graph 8: cdf of RCD/EXD Weibull



Discussion

For the appraisal of the probable duration of the recession we use the probability density functions of the adopted distributions. To this purpose we ask the probability for the variable to take with probability 95% a value less than a specific value x. In the last column of the following table 9 is shown that the recession duration RCD under the Fréchet distribution with probability 95% will last less than 49,52~50 month, while the same variable under the Burr distribution with same probability will last less than 51,91~52 month. Under the Fréchet or Burr distributions and given that the last recession started in October 2008, one can easily anticipate the end of the recession, always with 95% probability. On purpose we avoid to comment means and other higher moments since what is mainly of interest in the heavy-tailed distributions are the extreme values, let aside that some of these distributions do not have mean or variance or both (e.g. Cauchy) or they have only under specific values of the parameters (Pareto). It is remarkable the much more optimistic evaluation under the normal distribution which gives estimation for the duration of the recession (with probability 95%)

duration less than 42,99 ~ 43 month.

The practical consequence of the distribution of the EXD-RCD as Cauchy is that one cannot acquire information for the probable duration of the recession taking into consideration the duration of the immediately previous expansion duration. However, since the distribution of RCD/EXD as Weibull is not rejected, the value of the shape parameter of this distribution (α=0,93<1) imposes the idea that on the average the ratio of the recession to expansion duration is mildly diminishing.

Table 9: Values of X for P[X>x]=0,95

Variable	Distribution	Mean	StDev	Mode	x value at F(x)=0,95
RCD	Gumbel max	19,2979	14,4085	12,8133	46,1814
RCD	Weibull	21,4710	18,5850	6,0000	58,2435
RCD	Frechet	20,3611	41,5075	10,0283	49,5196
RCD	Gen.Extr.Value	19,2979	19,6586	10,4250	45,5639
RCD	Lognormal	19,4589	16,3655	9,1352	47,8064
RCD	Burr	21,3250	n/a	10,7563	51,9140
RCD	Pareto	208,0200	n/a	6,0000	110,0670
RCD	Cauchy			12,9779	40,0553
RCD	Levy			7,0051	1338,870
RCD	Normal	19,2979	14,4085	19,2979	42,9978
EXD-RCD	Cauchy			9,5928	87,0682
RCD/EXD	Weibull	0,7733	0,7555	0,0667	2,2804

Table 10: Goodness of fit of Frechet distribution to variable RCD

Goodness of Fit – Details RCD- Frechet (3P)					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,086724 0,841356 2				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 0,340789 1				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	5 3,89674 0,564377 7				
α Critical Value Reject?	0,2 7,28928 No	0,1 9,23636 No	0,05 11,0705 No	0,02 13,3882 No	0,01 15,0863 No
RCD- Frechet (3P)	α=2,15224 β=12,5866 γ=-0,513279				

Table 11: Goodness of fit of Burr distribution to variable RCD

Goodness of fit – Details RCD- Burr					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,085618 0,85196 1				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 0,350434 6				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 3,75323 0,440432 3				
α Critical Value	0,2 5,98862	0,1 7,77944	0,05 9,48773	0,02 11,6678	0,01 13,2767
Reject?	No	No	No	No	No
RCD- Burr	k=0,349938 α=5,18627 β=9,9694				

Table 12: Goodness of fit of Cauchy distribution to variable EXD-RCD

Goodness of Fit – Details EXD-RCD Cauchy					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,114852 0,527282 1				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 1,04505 12				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	3 2,77688 0,42732 3				
α Critical Value Reject?	0,2 4,64163 No	0,1 6,25139 No	0,05 7,81473 No	0,02 9,83741 No	0,01 11,3449 No
EXD-RCD- Cauchy	σ=12,2709 μ=9,5928				

Table 13: Goodness of fit of Weibull distribution to variable RCD/RCD

Goodness of Fit – Details RCD/EXD- Weibull (3P)					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,081678 0,887187 6				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 0,745287 19				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	5 2,99499 0,700759 22				
α	0,2	0,1	0,05	0,02	0,01
Critical Value	7,28928	9,23636	11,0705	13,3882	15,0863
Reject?	No	No	No	No	No
RCD/EXD- Weibull (3P)	α=0,93598 β=0,685558 γ=0,06667				

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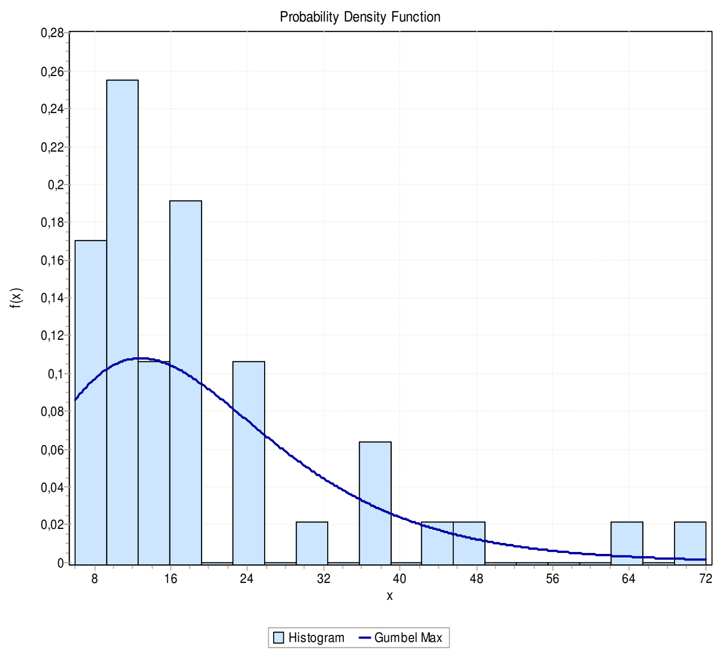
Appendix I: Graphs and Goodness of Fit of Distributions

Gumbel max distribution

Graph 9: pdf of RCD- Gumbel max



Graph 10: cdf of RCD- Gumbel max

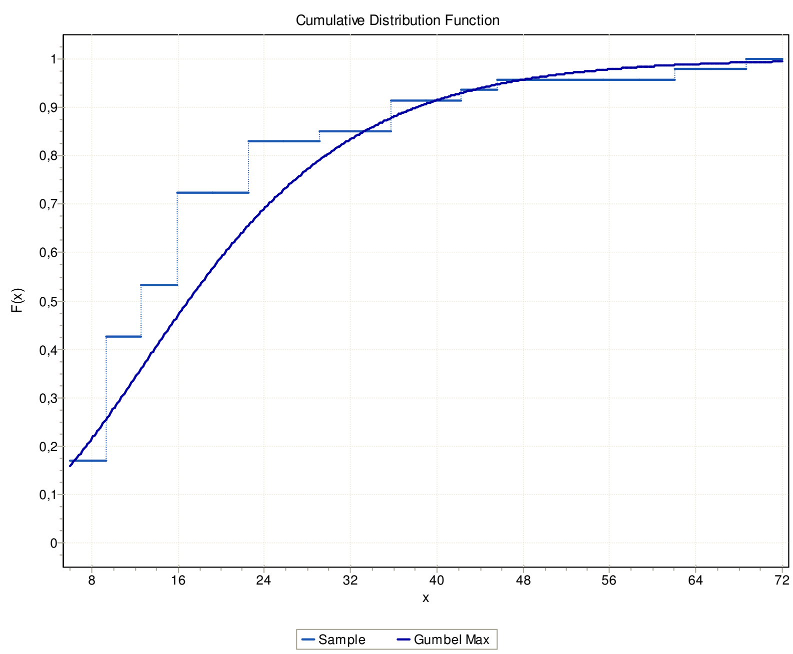
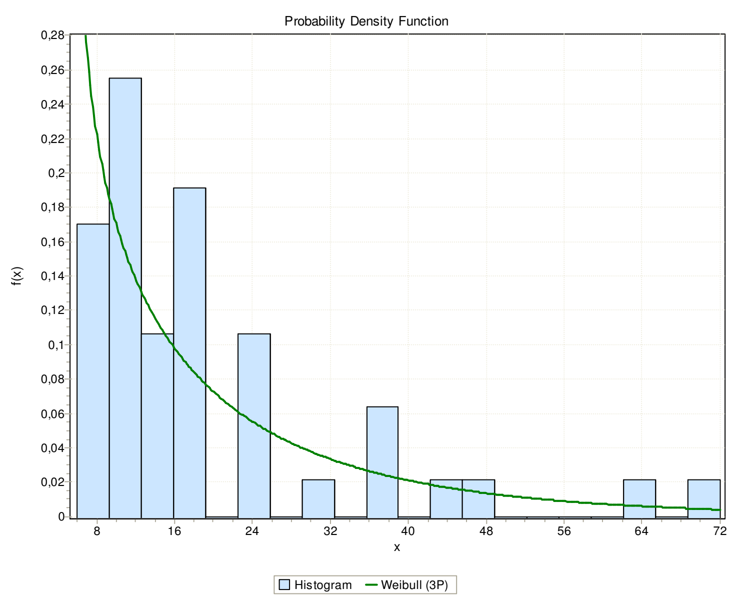


Table 14: Goodness of fit of Gumbel max distribution to variable RCD

Goodness of Fit – Details RCD- Gumbel Max					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,169653 0,118724 30				
α Critical Value Reject?	0,2 0,15295 Yes	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 2,1375 24				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 12,5816 0,013512 36				
α Critical Value Reject?	0,2 5,98862 Yes	0,1 7,77944 Yes	0,05 9,48773 Yes	0,02 11,6678 Yes	0,01 13,2767 No
RCD- Gumbel Max	σ=11,2343 μ=12,8133				

Weibull distribution

Graph 11: pdf of RCD- Weibull



Graph 12: cdf of RCD- Weibull

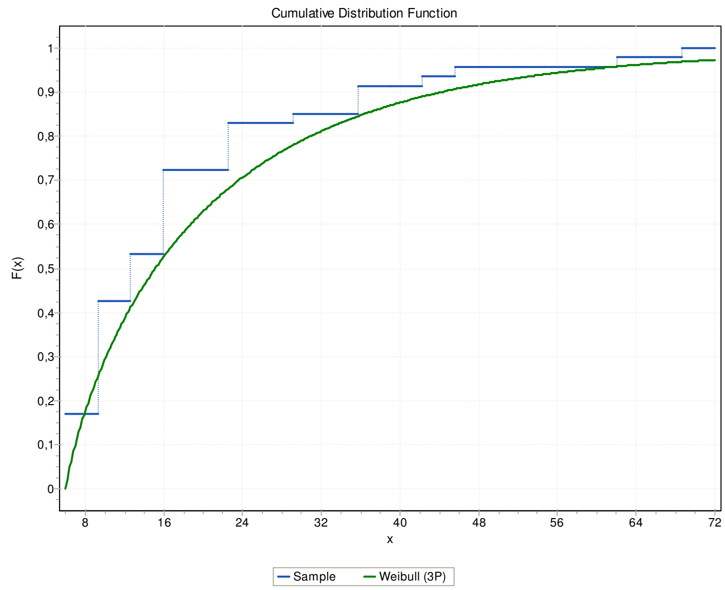
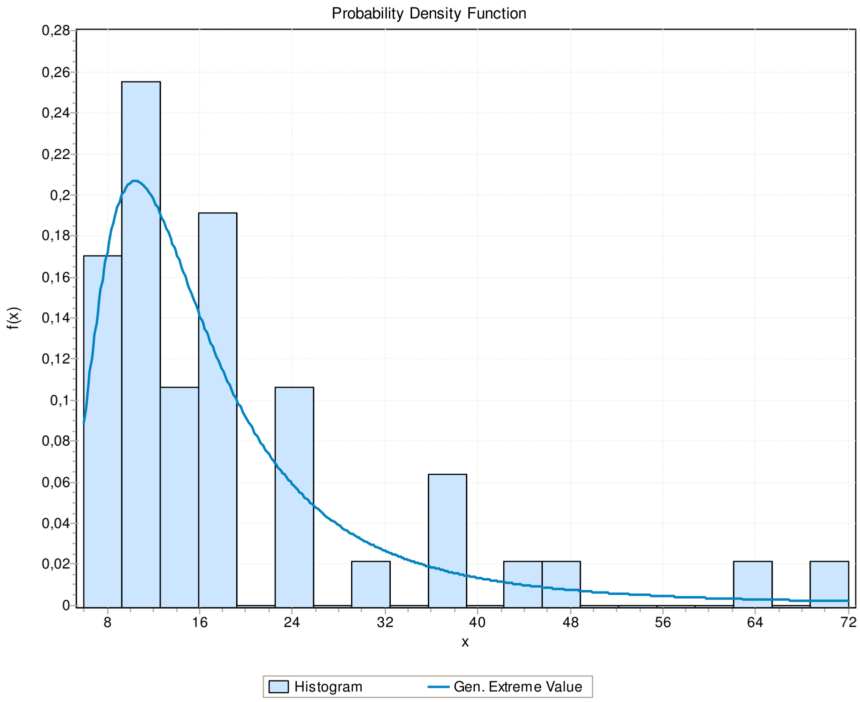


Table 15: Goodness of fit of Weibull distribution to variable RCD

Goodness of Fit – Details RCD- Weibull (3P)					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,124317 0,427293 20				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 2,73033 27				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 Yes	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 7,74516 0,101372 28				
α Critical Value Reject?	0,2 5,98862 Yes	0,1 7,77944 No	0,05 9,48773 No	0,02 11,6678 No	0,01 13,2767 No
RCD- Weibull (3P)	α=0,836771 β=14,0792 γ=6,0				

Generalized Extreme Value distribution (GEV)

Graph 13: pdf of RCD- Generalized Extreme Value



Graph 14: cdf of RCD- Generalized Extreme Value

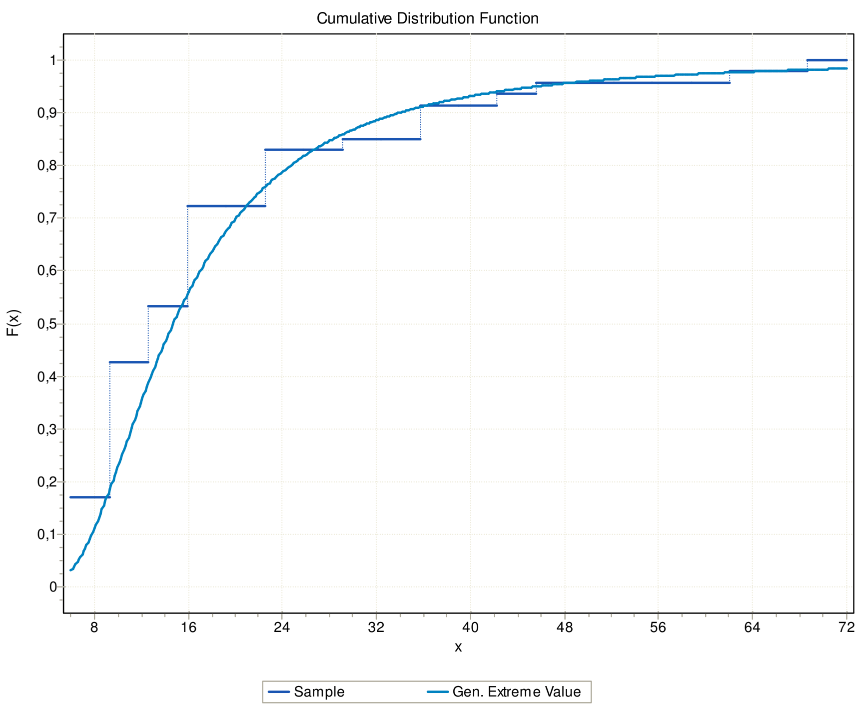
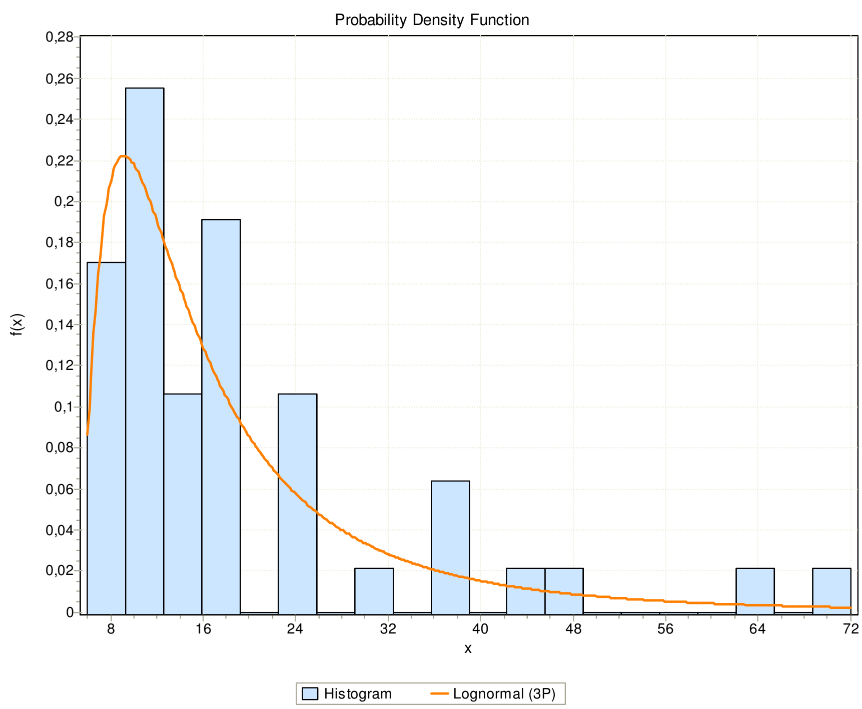


Table 16: Goodness of fit of Generalized Extreme Value distribution to variable RCD

Goodness of Fit – Details RCD- Gen. Extreme Value					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,099591 0,702309 11				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 0,375612 9				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	5 3,98898 0,551004 10				
α Critical Value Reject?	0,2 7,28928 No	0,1 9,23636 No	0,05 11,0705 No	0,02 13,3882 No	0,01 15,0863 No
RCD- Gen. Extreme Value	k=0,363864 σ=6,22587 μ=12,2519				

Lognormal distribution

Graph 15: pdf of RCY- Lognormal



Graph 16: cdf of RCY- Lognormal

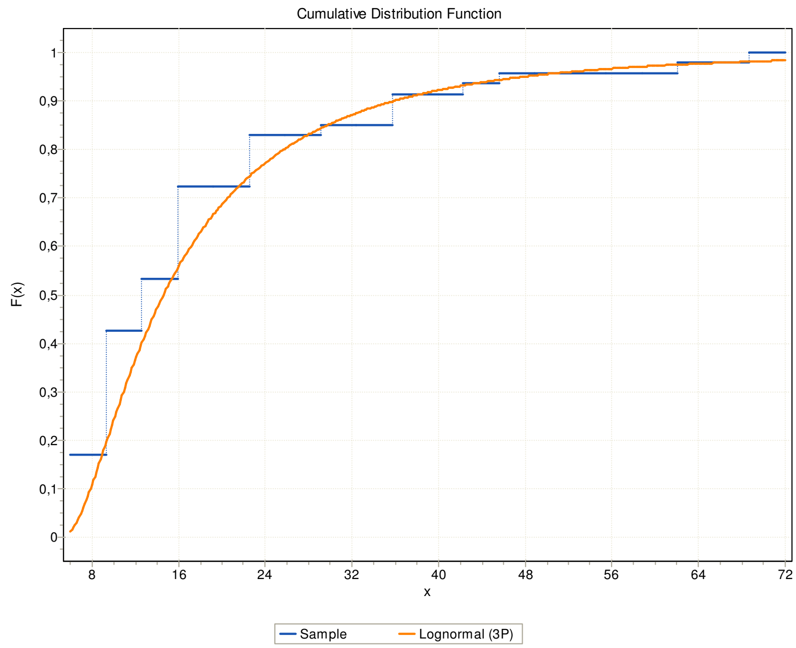
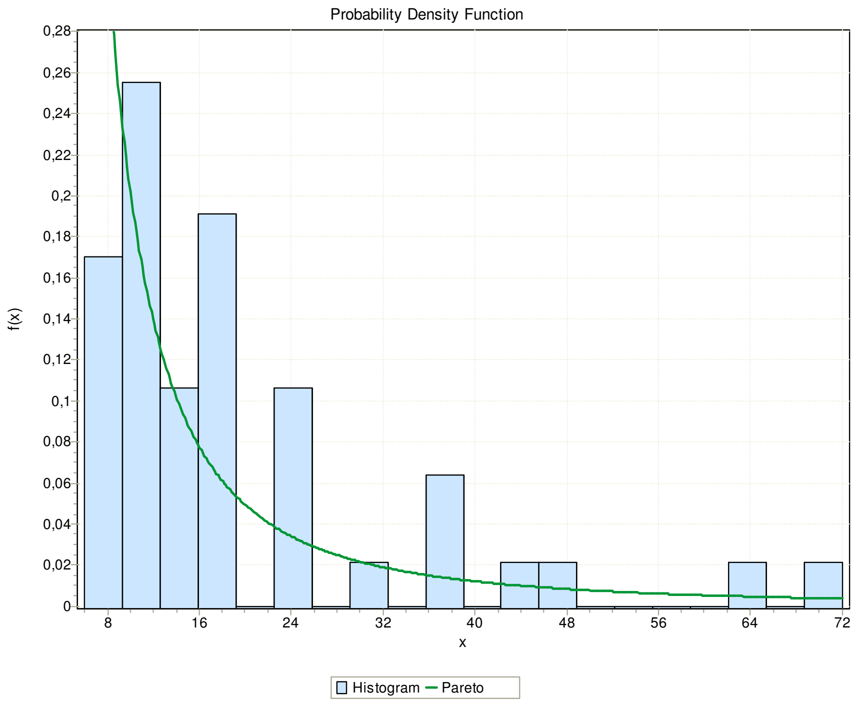


Table 17: Goodness of fit of Lognormal distribution to variable RCD

Goodness of fit – Details RCD- Lognormal (3P)					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,090839 0,799574 7				
α Critical Value Reject?	0,2 0,15295 No	0,1 0,17481 No	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 0,368626 8				
α Critical Value Reject?	0,2 1,37491 No	0,1 1,92862 No	0,05 2,50176 No	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	5 3,80358 0,578027 4				
α Critical Value Reject?	0,2 7,28928 No	0,1 9,23636 No	0,05 11,0705 No	0,02 13,3882 No	0,01 15,0863 No
RCD- Lognormal (3P)	σ=0,89585 μ=2,2899 γ=4,70988				

Pareto distribution

Graph 17: pdf of RCD- Pareto



Graph 18: cdf of RCD- Pareto

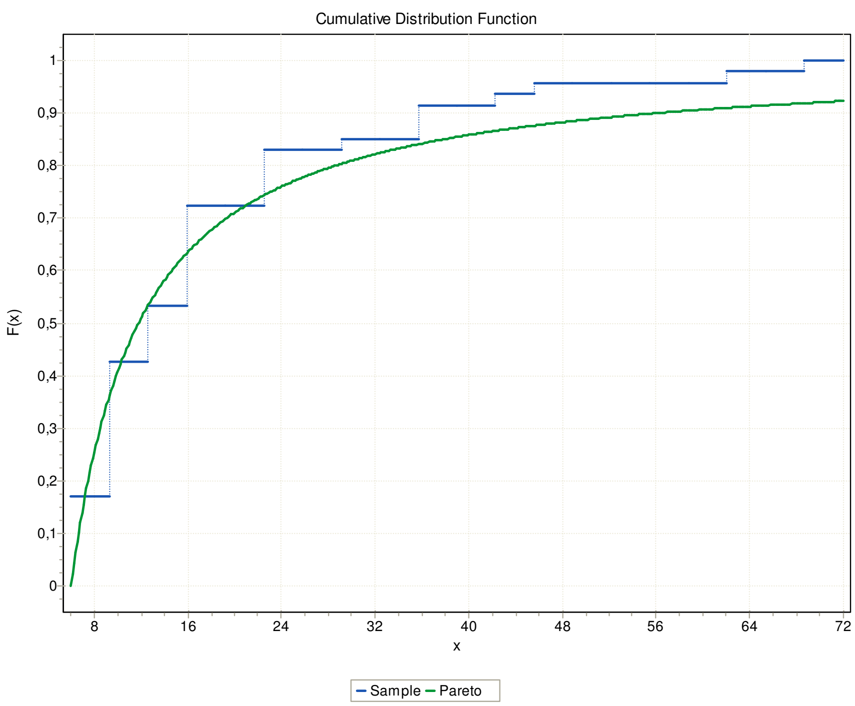
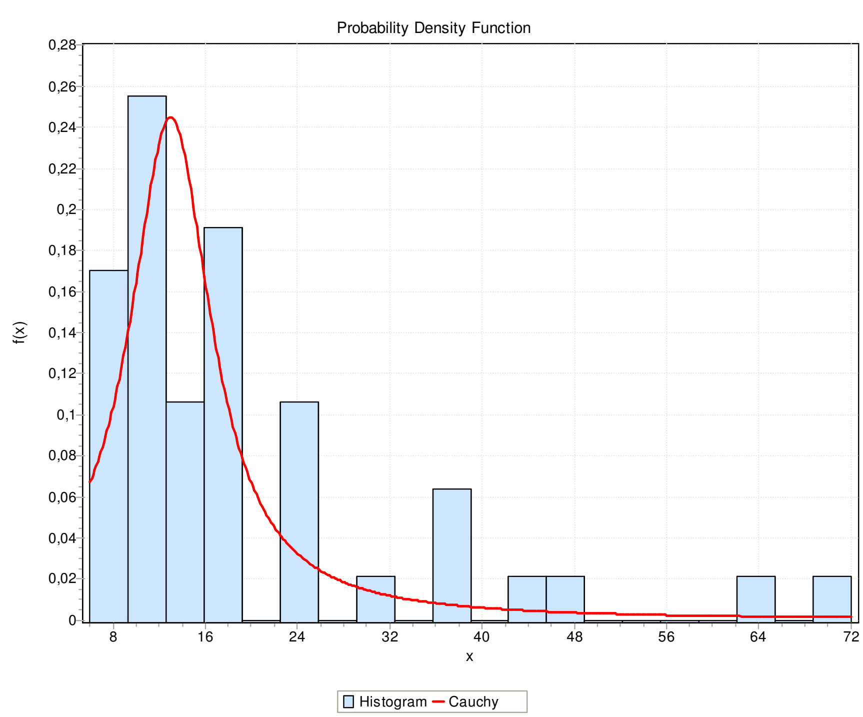


Table 18: Goodness of fit of Pareto distribution to variable RCD

Goodness of fit – Details **RCD-Pareto**					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,238822 0,007644 40				
α Critical Value Reject?	0,2 0,15295 Yes	0,1 0,17481 Yes	0,05 0,1942 Yes	0,02 0,21715 Yes	0,01 0,23298 Yes
Anderson-Darling					
Sample Size Statistic Rank	47 6,62536 46				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 Yes	0,02 3,2892 Yes	0,01 3,90742 Yes
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	3 10,4333 0,01522 32				
α Critical Value Reject?	0,2 4,64163 Yes	0,1 6,25139 Yes	0,05 7,81473 Yes	0,02 9,83741 Yes	0,01 11,3449 No
RCD- Pareto	α=1,0297 β=6				

Cauchy distribution

Graph 19: pdf of RCD- Cauchy



Graph 20: cdf of RCD- Cauchy

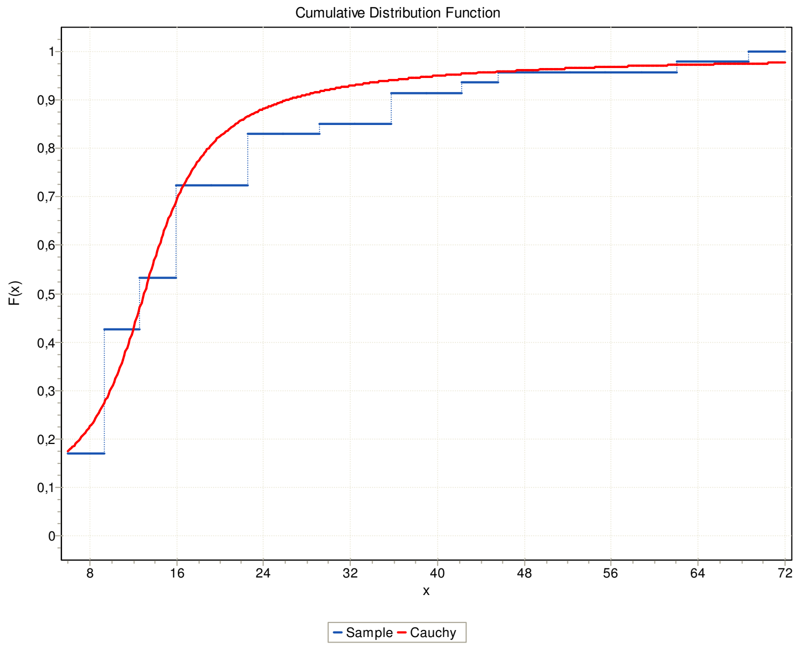
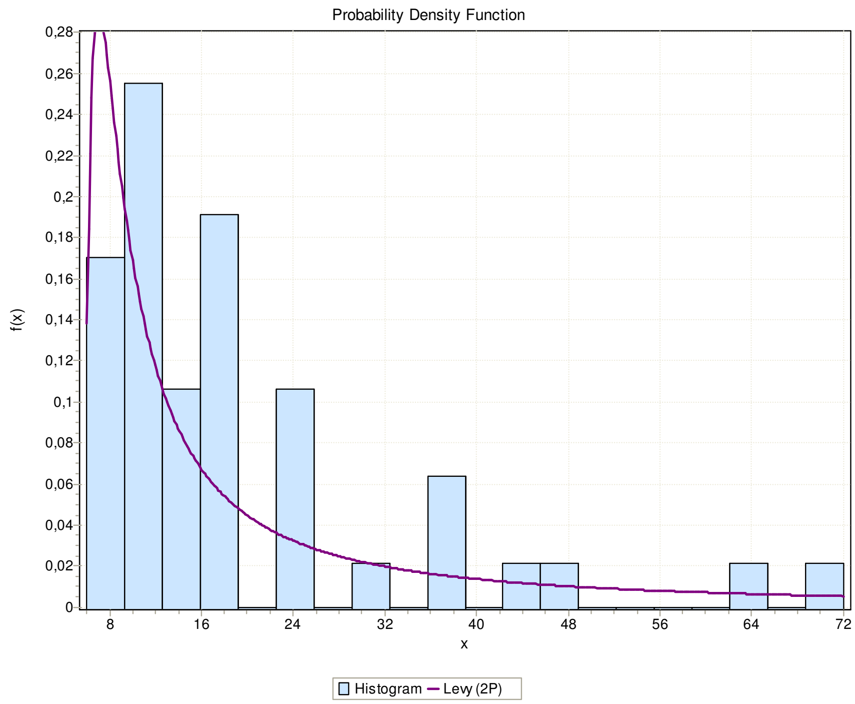


Table 19: Goodness of fit of Cauchy distribution to variable RCD

Goodness of fit – Details **RCD- Cauchy**					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,179279 0,085775 32				
α Critical Value Reject?	0,2 0,15295 Yes	0,1 0,17481 Yes	0,05 0,1942 No	0,02 0,21715 No	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 3,15149 32				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 Yes	0,02 3,2892 No	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 7,61665 0,106674 27				
α Critical Value Reject?	0,2 5,98862 Yes	0,1 7,77944 No	0,05 9,48773 No	0,02 11,6678 No	0,01 13,2767 No
RCD- Cauchy	σ=4,28864 μ=12,9779				

Levy distribution

Graph 21: pdf of RCD- Levy



Graph 22: cdf of RCD- Levy

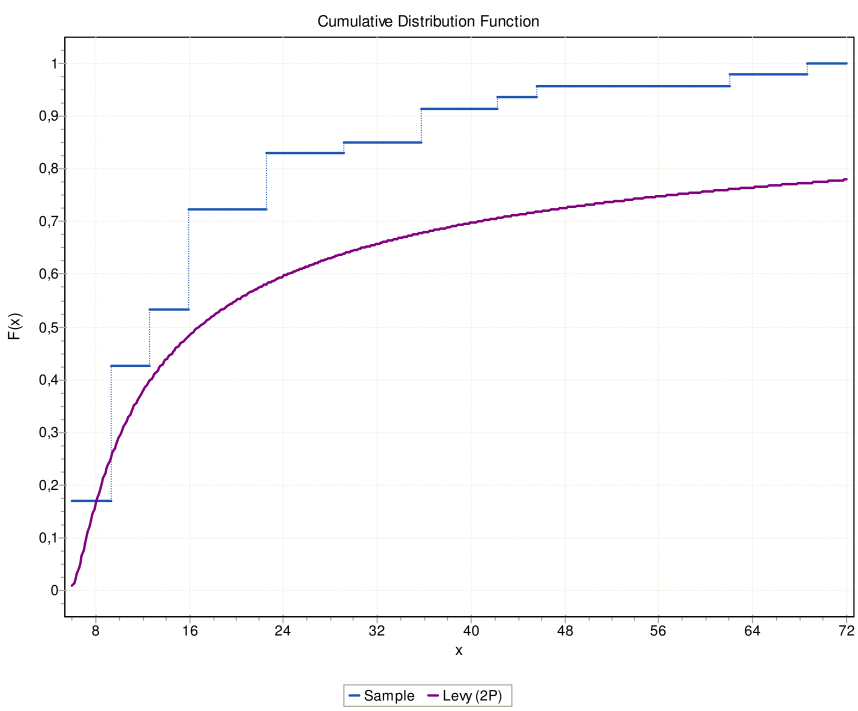
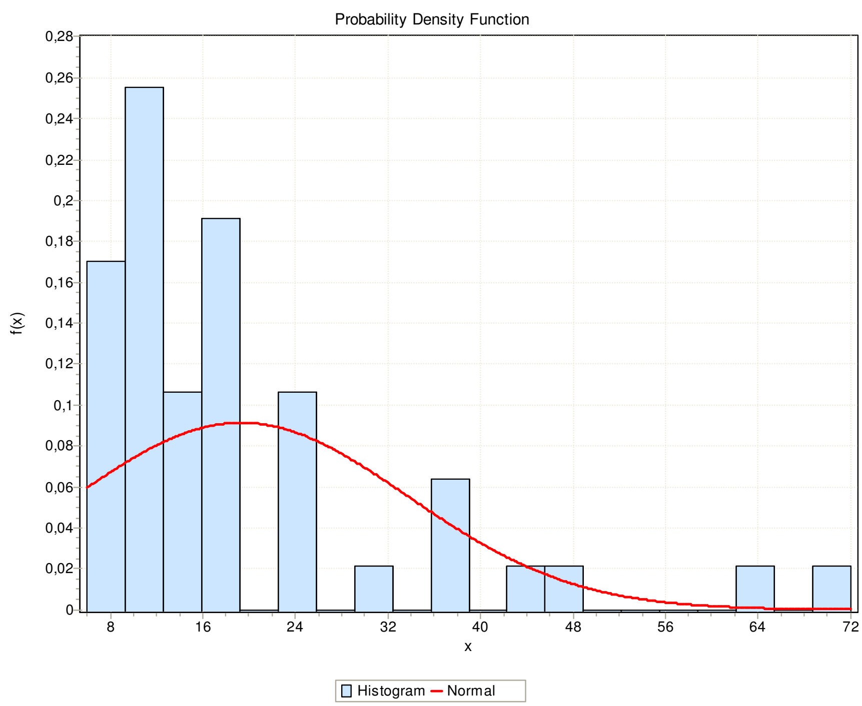


Table 20: Goodness of fit of Levy distribution to variable RCD

Goodness of fit – Details RCD- Levy (2P)					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,232945 0,010015 37				
α Critical Value Reject?	0,2 0,15295 Yes	0,1 0,17481 Yes	0,05 0,1942 Yes	0,02 0,21715 Yes	0,01 0,23298 No
Anderson-Darling					
Sample Size Statistic Rank	47 4,10135 40				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 Yes	0,02 3,2892 Yes	0,01 3,90742 Yes
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 19,796 5,47901E-4 42				
α Critical Value Reject?	0,2 5,98862 Yes	0,1 7,77944 Yes	0,05 9,48773 Yes	0,02 11,6678 Yes	0,01 13,2767 Yes
RCD- Levy (2P)	σ=5,24397 γ=5,25712				

Normal distribution

Graph 23: pdf of RCD- Normal



Graph 24: cdf of RCD- Normal

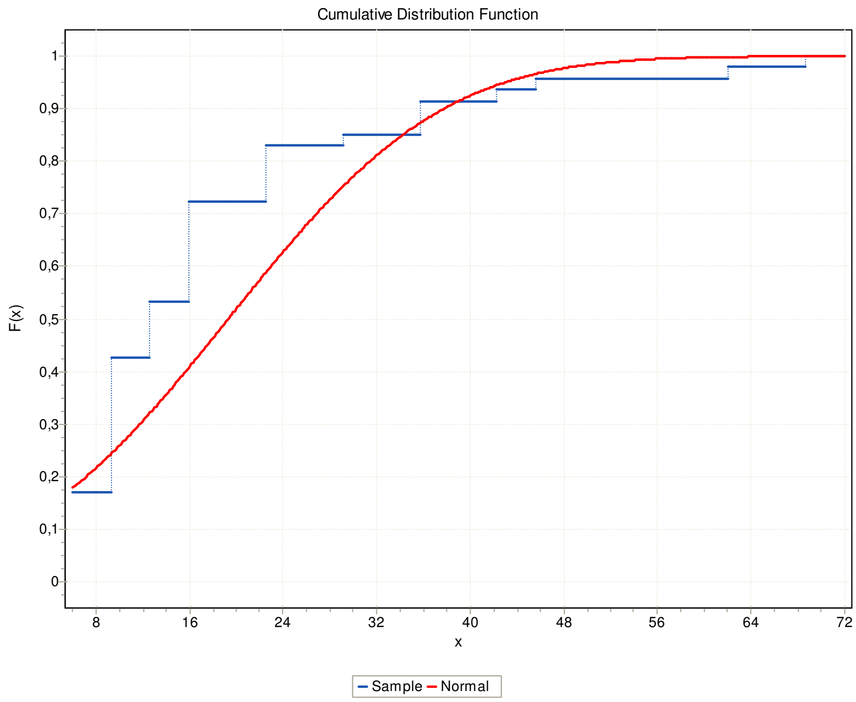


Table 21: Goodness of fit of Normal distribution to variable RCD

Goodness of fit – Details **RCD- Normal**					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	47 0,238015 0,007936 39				
α Critical Value Reject?	0,2 0,15295 Yes	0,1 0,17481 Yes	0,05 0,1942 Yes	0,02 0,21715 Yes	0,01 0,23298 Yes
Anderson-Darling					
Sample Size Statistic Rank	47 3,78286 36				
α Critical Value Reject?	0,2 1,37491 Yes	0,1 1,92862 Yes	0,05 2,50176 Yes	0,02 3,2892 Yes	0,01 3,90742 No
Chi-Squared					
Deg. of freedom Statistic P-Value Rank	4 4,40281 0,354227 12				
α Critical Value Reject?	0,2 5,98862 No	0,1 7,77944 No	0,05 9,48773 No	0,02 11,6678 No	0,01 13,2767 No
RCD- Normal	σ=14,4085 μ=19,2979				

APPENDIX II - The Data

Table 22: List of Recessions in the USA 1791-2008

Name	Dates	Recession duration	Time since previous recession	EXD (Mo)	RCD (Mo)	CYD (Mo)
Panic of 1797	1796–1799	~3 years	~6 years	72	36	108
1802–1804 recession	1802–1804	~2 years	~3 years	36	24	60
Depression of 1807	1807–1810	~3 years	~3 years	36	36	72
1812 recession	1812	~6 months	~18 months	18	6	24
1815–21 depression	1815–1821	~6 years	~3 years	36	72	108
1822–1823 recession	1822–1823	~1 year	~1 year	12	12	24
1825–1826 recession	1825–1826	~1 year	~2 years	24	12	36
1828–1829 recession	1828–1829	~1 year	~2 years	24	12	36
1833–34 recession	1833–1834	~1 year	~4 years	48	12	60
Panic of 1837	1836–1838	~2 years	~2 years	24	24	48
Depression of 1839–43	Late 1839 – Late 1843	~4 years	~1 year	12	48	60
1845–46 recession	1845 –Late 1846	~1 year	~2 years	24	12	36
1847–48 recession	Late 1847 – Late 1848	~1 year	~1 year	12	12	24
1853–54 recession	1853 – Dec 1854	~1 year	~5 years	60	12	72
Panic of 1857	June 1857 – Dec 1858	1 year 6 months	2 years 6 months	30	18	48
1860–61 recession	Oct 1860 – June 1861	8 months	1 year 10 months	22	8	30
1865–67 recession	April 1865 –Dec 1867	2 years 8 months	3 years 10 months	46	32	78
1869–70 recession	June 1869 – Dec 1870	1 year 6 months	1 year 6 months	18	18	36
Panic of 1873 – Long Depress	Oct 1873 – Mar 1879	5 years 5 months	2 years 10 months	34	65	99
1882–85 recession	Mar 1882 – May 1885	3 years 2 months	3 years	36	38	74
1887–88 recession	Mar 1887 –April 1888	1 year 1 months	1 year 10 months	22	13	35
1890–91 recession	July 1890 – May 1891	10 months	1 year 5 months	20	10	30
Panic of 1893	Jan 1893 – June 1894	1 year 5 months	1 year 8 months	18	17	35
Panic of 1896	Dec 1895 – June 1897	1 year 6 months	1 year 6 months	18	18	36
1899–1900 recession	June 1899 – Dec 1900	1 year 6 months	2 years	24	18	42
1902–04 recession	Sep 1902 – Aug 1904	1 year 11 months	1 year 9 months	21	23	44
Panic of 1907	May 1907 –June 1908	1 year 1 month	2 years 9 months	33	13	46
Panic of 1910–1911	Jan 1910 – Jan 1912	2 years	1 year 7 months	19	24	43
Recession of 1913–1914	Jan 1913 – Dec 1914	1 year 11 months	1 year	12	23	35
Post-World War I recess	Aug 1918 –March 1919	7 months	3 years 8 months	42	7	49
Depression of 1920–21	Jan 1920 – July 1921	1 year 6 months	10 months	10	18	28
1923–24 recession	May 1923 –June 1924	1 year 2 months	2 years	24	14	38
1926–27 recession	Oct 1926 – Nov 1927	1 year 1 month	2 years 3 months	27	13	40
Great Depression	Aug 1929 – Mar 1933	3 years 7 months	1 year 9 months	21	43	64
Recession of 1937	May 1937 –June 1938	1 year 1 month	4 years 2 months	50	13	63
Recession of 1945	Feb–Oct 1945	8 months	6 years 8 months	80	8	88
Recession of 1949	Nov 1948 – Oct 1949	11 months	3 years 1 month	37	11	48
Recession of 1953	July 1953 – May 1954	10 months	3 years 9 months	45	10	55

Recession of 1958	Aug 1957 –April 1958	8 months	3 years 3 months	39	8	47
Recession of 1960–61	Apr 1960 – Feb 1961	10 months	2 years	24	10	34
Recession of 1969–70	Dec 1969 – Nov 1970	11 months	8 years 10 months	106	11	117
1973–75 recession	Nov 1973 – Mar 1975	1 year 4 months	3 years	36	16	52
1980 recession	Jan–July 1980	6 months	4 years 10 months	58	6	64
Early 1980s recession	July 1981 – Nov 1982	1 year 4 months	1 year	12	16	28
Early 1990s recession	July 1990 – Mar 1991	8 months	7 years 8 months	92	8	100
Early 2000s recession	Mar–Nov 2001	8 months	10 years	120	8	128
Late 2000s recession	Dec 2007- July 2009	19 months	6 years 1 month	73	19	92

Source: National Bureau of Economic Research