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Existence of Arch Effect in Consumer Goods Deflator the Greek Case 1960-1994

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Abstract

Heteroscedasticity is a typical phenomenon in economic and financial time series. The variance of the series changes with time. This effect makes the series *ipso facto* non-stationary and deprives the investigator of the mathematical and statistical apparatus tailored for stationary time series. Given this situation, the investigator is obliged to consider non-linear stochastic models- the most popular of which are the (G)ARCH ones. In this study we give the theoretical background of the ARCH model and an econometric application of the model to the Greek consumer goods deflator. The study gives evidence that ARCH effects are present in the investigated time series.

Keywords: Arch Effect, Greek Consumer Goods Deflator, Heteroscedasticity.

Introduction

Although there is no general agreement for the meaning of a linear stochastic model there is a strong tradition to consider a stochastic model as linear if it is linear in the parameters. In this sense an autoregressive process of p order AR(p) is considered as linear. This conception of linearity results from the convenience obtained when applying expectations to a stochastic expression, for expectations are linear transformations. Several families of non-linear models have been introduced in the attempt to deal with non-linear models as non-linear autoregressive processes, threshold models, bilinear models (Priestly, 1988) and chaotic models (Mandelbot, 1995). Especially for cases of changing variance classes of models described as Autoregressive Conditional Heteroscedasticity models (ARCH models) and Generalized ARCH models (GARCH models) have been introduced. These models do not generally obtain better point forecasts but they can help to obtain better estimates of the local variance, which allows construction of more reliable prediction intervals and better risk assessment. This is important for financial time series, which generally exhibit sudden changes of variance (volatility). More specifically, the ARCH models are martingale differences and hence knowledge of the variance in time t σ_t^2 does not lead to better point forecast in time t+s. Moreover, as is it is shown in the appropriate section of this study, even with constant unconditional variance, the conditional variance can change with time. The last consideration renders the use of (G)ARCH models indispensable even in cases of stationary time series.

In our study we investigate presence of ARCH effect in the annual rate of change in the consumer goods deflator. In section 2 we define the data, the variables, their descriptions and symbols. In section 3 we present the theoretical background of the ARCH modeling, its application to our data and a discussion of the findings. The section 4- discussion of the findings- contains a summary of some drawbacks concerning the model the application and the findings.

The Data

The data consists of 35 annual measurements of the consumer goods deflator (CGD) from 1960 through 1994 (source: OECD). The data is shown in the column titled D. For all calculations, graphs and results we have employed the program STATISTICA. For easy reference, we have embedded all tables and graphs in this text. We have on purpose limited the series up to 1994 since recent economic data are in the most of times not definite and they involve political considerations and conflict of interest. In the following tables 1 and 2 we submit the list of all variables and the list of tables and graphs relevant to this study, accordingly.

Table 1: List of variables, symbols and descriptions

Symbol	Mathematical definition	Description
D	D_t	Consumer goods deflator in time t
RD	$RD_t = (D_t - D_{t-1}) / D_{t-1})$	Rate of change of D from time t-1 to time t
RD_1	RD_{t-1}	Rate of change of D from time t-2 to time t-1
U	u_t	Residual from simple regression of RD_t to RD_{t-1}
U2	u_t^2	Squared residual in time t
U2_1	u_{t-1}^2	Squared residual with lag 1
U2_2	u_{t-2}^2	Squared residual with lag 2
U2_3	u_{t-3}^2	Squared residual with lag 3
U2_4	u_{t-4}^2	Squared residual with lag 4
ACF	Autocorrelation function	Autocorrelation function of the residuals u_t
PACF	Partial autocorrelation function	Partial autocorrelation function of the residuals u_t

Table 2: List of statistica files for tables and graphs

File type	File serial number	File name	File content
Table	3	ARCH EFFECT-DATA	Original data, derived series, residuals
Table	4	Regression RD to RD_1	Regression results in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$
Table	5	ARCH(1)	Regression results in ARCH(1) model
Table	6	ARCH(2)	Regression results in ARCH(2) model
Table	7	ARCH(3)	Regression results in ARCH(3) model
Table	8	ARCH(4)	Regression results in ARCH(4) model
Table	9	-	Regression summary
Graph	1	Graph 1-Deflator (D)	Graph of the deflator
Graph	2	Graph 2-Squared regression residuals	Squared regression residuals in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$
Graph	3	Graph 3-Aggregated squared regression residuals	Aggregated squared regression residuals in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$
Graph	4	Graph 4-Regression residuals	Regression residuals in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$
Graph	5	Graph 2-Aggregated deflator	Aggregated graph of the deflator
Graph	6	Graph 4-Aggregated RD	Aggregated graph of RD
Graph	7	Graph 3-Change rate in D	Graph of RD
Graph	8	Graph 8-ACF of regression residuals	Autocorrelation function of the regression residuals in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$
Graph	9	Graph 9- PACF of regression residuals	Partial autocorrelation function of the regression residuals in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$

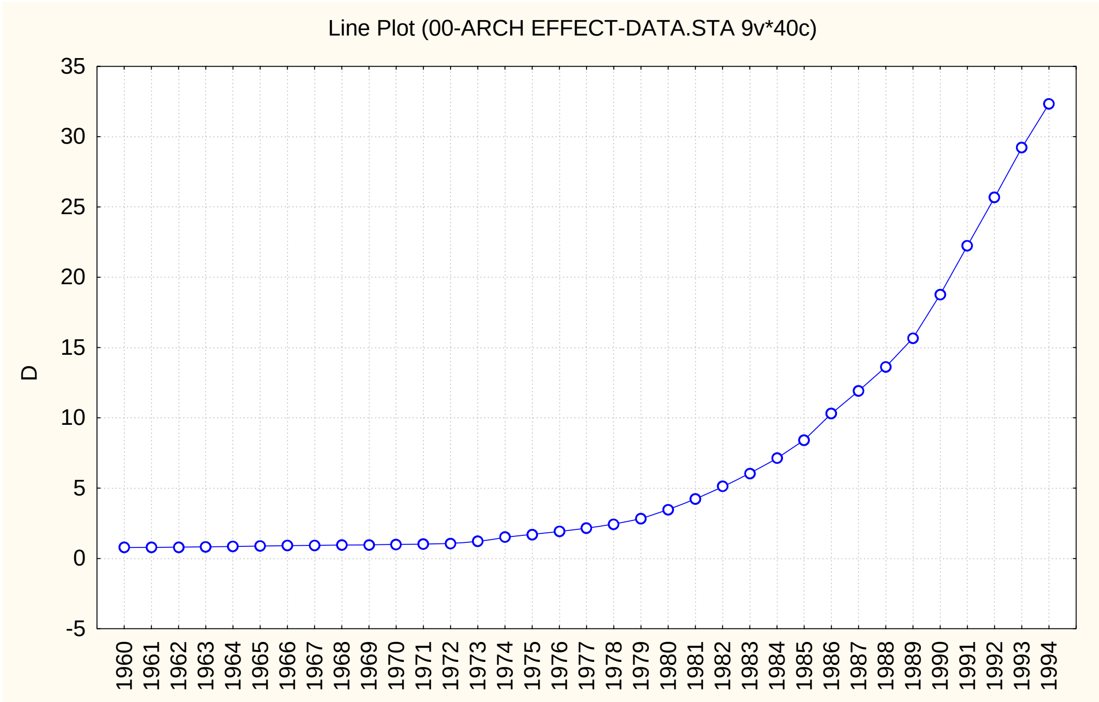
In the following table 3 are shown the original data (YEAR, D), the derived series and the residuals

Table 3: Arch effect-data original data, derived series and residuals

YEAR	D	RD	RD_1	U	U2	U2_1	U2_2	U2_3	U2_4
1960	0,783								
1961	0,792	0,011							
1962	0,802	0,013	0,011	-0,020	0,000				
1963	0,829	0,034	0,013	0,001	0,000	0,000			
1964	0,847	0,022	0,034	-0,029	0,001	0,000	0,000		
1965	0,886	0,046	0,022	0,005	0,000	0,001	0,000	0,000	
1966	0,917	0,035	0,046	-0,026	0,001	0,000	0,001	0,000	0,000
1967	0,934	0,019	0,035	-0,033	0,001	0,001	0,000	0,001	0,000
1968	0,941	0,007	0,019	-0,031	0,001	0,001	0,001	0,000	0,001
1969	0,970	0,031	0,007	0,002	0,000	0,001	0,001	0,001	0,000
1970	1,000	0,031	0,031	-0,017	0,000	0,000	0,001	0,001	0,001
1971	1,029	0,029	0,031	-0,019	0,000	0,000	0,000	0,001	0,001
1972	1,062	0,032	0,029	-0,015	0,000	0,000	0,000	0,000	0,001
1973	1,222	0,151	0,032	0,101	0,010	0,000	0,000	0,000	0,000
1974	1,509	0,235	0,151	0,087	0,008	0,010	0,000	0,000	0,000
1975	1,701	0,127	0,235	-0,091	0,008	0,008	0,010	0,000	0,000
1976	1,930	0,135	0,127	0,006	0,000	0,008	0,008	0,010	0,000
1977	2,160	0,119	0,135	-0,016	0,000	0,000	0,008	0,008	0,010
1978	2,436	0,128	0,119	0,006	0,000	0,000	0,000	0,008	0,008
1979	2,838	0,165	0,128	0,036	0,001	0,000	0,000	0,000	0,008
1980	3,463	0,220	0,165	0,060	0,004	0,001	0,000	0,000	0,000
1981	4,242	0,225	0,220	0,019	0,000	0,004	0,001	0,000	0,000
1982	5,119	0,207	0,225	-0,003	0,000	0,000	0,004	0,001	0,000
1983	6,047	0,181	0,207	-0,014	0,000	0,000	0,000	0,004	0,001
1984	7,127	0,179	0,181	0,005	0,000	0,000	0,000	0,000	0,004
1985	8,422	0,182	0,179	0,010	0,000	0,000	0,000	0,000	0,000
1986	10,301	0,223	0,182	0,049	0,002	0,000	0,000	0,000	0,000
1987	11,920	0,157	0,223	-0,051	0,003	0,002	0,000	0,000	0,000
1988	13,619	0,143	0,157	-0,011	0,000	0,003	0,002	0,000	0,000
1989	15,672	0,151	0,143	0,009	0,000	0,000	0,003	0,002	0,000
1990	18,776	0,198	0,151	0,050	0,002	0,000	0,000	0,003	0,002
1991	22,231	0,184	0,198	-0,004	0,000	0,002	0,000	0,000	0,003
1992	25,677	0,155	0,184	-0,021	0,000	0,000	0,002	0,000	0,000
1993	29,210	0,138	0,155	-0,014	0,000	0,000	0,000	0,002	0,000
1994	32,335	0,107	0,138	-0,030	0,001	0,000	0,000	0,000	0,002
1995			0,107			0,001	0,000	0,000	0,000
1996							0,001	0,000	0,000
1997								0,001	0,000
1998									0,001
1999									

In the following graph 1 is shown the time series of the original data (D, Consumer Goods Deflator)

Graph 1: Deflator (D)



Statistical Analysis
The Theoretical Part

Consider the following AR(1) scheme

$$u_t=\rho u_{t-1}+\varepsilon_t \quad (1)$$

which we can, without loss of generality, consider as stationary ($|\rho|<1$), with ε_t uncorrelated identically distributed stochastic variables with zero mean and variance σ_ε^2. The above scheme may be considered as representing the residuals of a regression, say X_t to X_{t-1}. Using the symbols E for expectations and V for variance and exploiting the well-known properties of the expectation and the variance for uncorrelated variables, we can easily obtain the results:

$E(u_t)=\rho E(u_{t-1})+E(\varepsilon_t)$ and $V(u_t)=\rho^2 V(u_{t-1})+V(\varepsilon_t)$. The last two expressions give

$$E(u_t)=0 \quad (2)$$

$$V(u_t)=\sigma_\varepsilon^2/(1-\rho^2) \quad (3)$$

But although the unconditional expectation of u_t is constant (zero in this case), its conditional expectation is not:

$$E(u_t|u_{t-1})=E(\rho u_{t-1}+\varepsilon_t)|u_{t-1})=E(\rho u_{t-1}|u_{t-1})+E(\varepsilon_t|u_{t-1})=\rho u_{t-1}+0=\rho u_{t-1}. \quad (4)$$

However, the conditional variance of the scheme is constant:

$$V(u_t|u_{t-1})=E[(\rho u_{t-1}+\varepsilon_t)^2|u_{t-1}]-(\rho u_{t-1})^2= (\rho u_{t-1})^2+V(\varepsilon_t^2)-(\rho u_{t-1})^2=\sigma_\varepsilon^2 \quad (5)$$

The above results indicate that an ordinary linear autoregressive scheme AR(p) cannot help in situations in which the variance is variable. One way to sort this out is to use for u a model, which allows for:

- Constant unconditional mean
- Constant conditional mean of
- Uncorrelated u's
- Constant unconditional variance
- Changing conditional variance

This model exists and is of the (non-linear) form

$$u_t=\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2} \quad (6)$$

α_0, α_1 non-negative numbers and ε_t uncorrelated, identically distributed stochastic variables, uncorrelated with all u's, with zero mean and unit variance (the last two requirements are not obligatory since any stochastic variable can be transformed to a new variable with zero mean and unit variance). The above model is called autoregressive conditional heteroscedasticity model of order 1, and it is denoted by the symbol ARCH(1). In the general case where u depends on its p previous values model it is called ARCH of order p and is denoted by

$$\text{ARCH(p)}: u_t=\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2+\alpha_2 u_{t-2}^2+\ldots \ldots .+\alpha_p u_{t-p}^2)^{1/2} \quad (7)$$

Let us prove that the model (6) satisfies the wished propertied 1/ through 4/ above:

For 1.

$$E(u_t)=E[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]=E(\varepsilon_t)E(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}=0xE(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}=0$$

For 2.

$$E(u_t|u_{t-1})=E[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}|u_{t-1}]=E[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]=E(\varepsilon_t)E(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]=0$$

For 3.

$$E(u_t u_{t-1})=E\{[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}][\varepsilon_{t-1}(\alpha_0+\alpha_1 u_{t-2}^2)^{1/2}]\}=E[\varepsilon_t\varepsilon_{t-1}(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}(\alpha_0+\alpha_1 u_{t-2}^2)^{1/2}]=$$
$$E(\varepsilon_t\varepsilon_{t-1})E[(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}(\alpha_0+\alpha_1 u_{t-2}^2)^{1/2}]=0xE[(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}(\alpha_0+\alpha_1 u_{t-2}^2)^{1/2}]=0$$

For 4.

$$V(u_t)=V[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]=E[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]^2=E(\varepsilon_t^2)E(\alpha_0+\alpha_1 u_{t-1}^2)=\sigma_\varepsilon^2[\alpha_0+\alpha_1 E(u_{t-1}^2)]$$
$$=\sigma_\varepsilon^2[\alpha_0+\alpha_1 V(u_{t-1})]=\sigma_\varepsilon^2[\alpha_0+\alpha_1 V(u_t)] \Rightarrow V(u_t)=\alpha_0\sigma_\varepsilon^2/(1-\alpha_1\sigma_\varepsilon^2)=\alpha_0/(1-\alpha_1)$$

For 5

$$V(u_t|u_{t-1})=E\{[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]^2|u_{t-1}\}-[E(u_t|u_{t-1})]^2 = E\{[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]^2|u_{t-1}\}-0=$$
$$E[\varepsilon_t(\alpha_0+\alpha_1 u_{t-1}^2)^{1/2}]^2=E[\varepsilon_t^2(\alpha_0+\alpha_1 u_{t-1}^2)=(E\varepsilon_t^2)(\alpha_0+\alpha_1 u_{t-1}^2)=\alpha_0+\alpha_1 u_{t-1}^2$$

One can easily show that the conditional variance for an ARCH(p) process is

$$\sigma_t^2=\alpha_0+\alpha_1 u_{t-1}^2+\alpha_2 u_{t-2}^2+\ldots \ldots .+\alpha_p u_{t-p}^2 \quad (8)$$

The rationale behind the use of an ARCH model is that if not all parameters α_1, α_2,.. α_p are simultaneously zeros, then the model can be considered as offering a plausible pattern of changing variance.

Hence the problem can be formalized as follows:

Step 1- Obtain the residuals u_t

Step 2-Proceed to the regressions in the models

$$u_t^2=\alpha_0+\alpha_1 u_{t-1}^2+\alpha_2 u_{t-2}^2+\ldots \ldots .+\alpha_p u_{t-p}^2 \quad p=1,2,... \quad (9)$$

Step 3-Test the null hypothesis that all α's are zero at a given significance level. If the test does not reject the H_0, then the ARCH model cannot be a candidate for explaining the changing variance (strictly speaking the rejection concerns rejection of ARCH in case that the variance is really changing). Rejection of H_0 advocates for the ARCH model as a good candidate for the description of changing variance (if it is really changing).

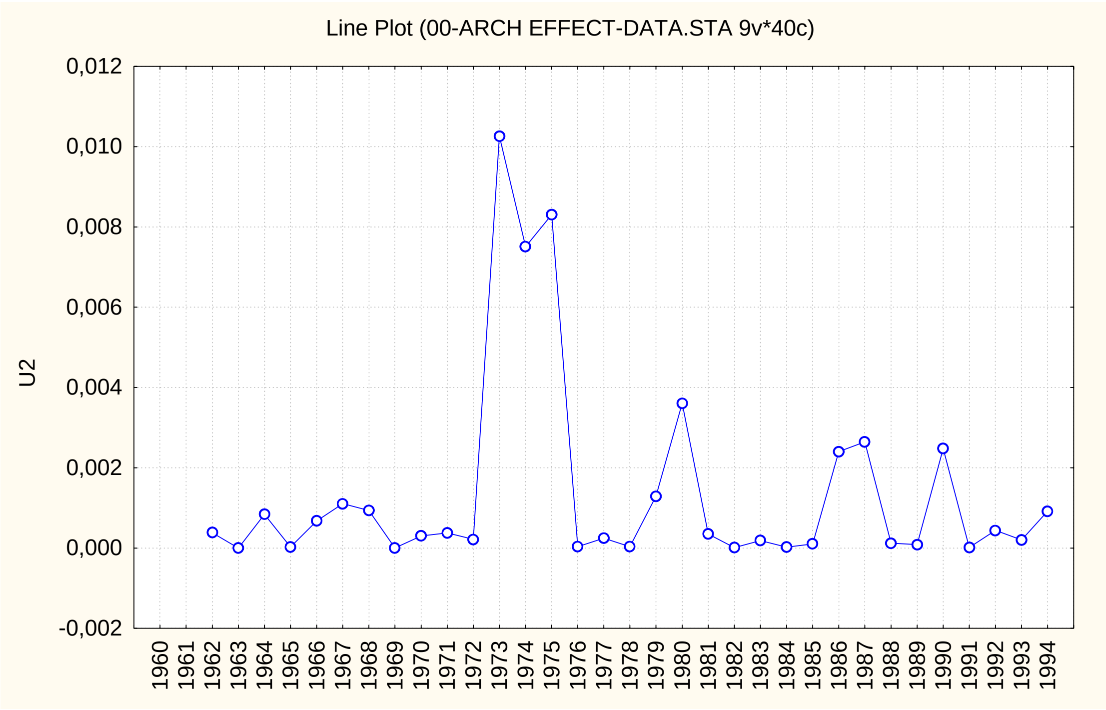
In the following table 4 are shown the regression results in model $RD_t=\alpha_0+\alpha_1 RD_{t-1}$

Table 4: Regression rd to rd-1 regression results in model $rd_t=\alpha_0+\alpha_1 rd_{t-1}$

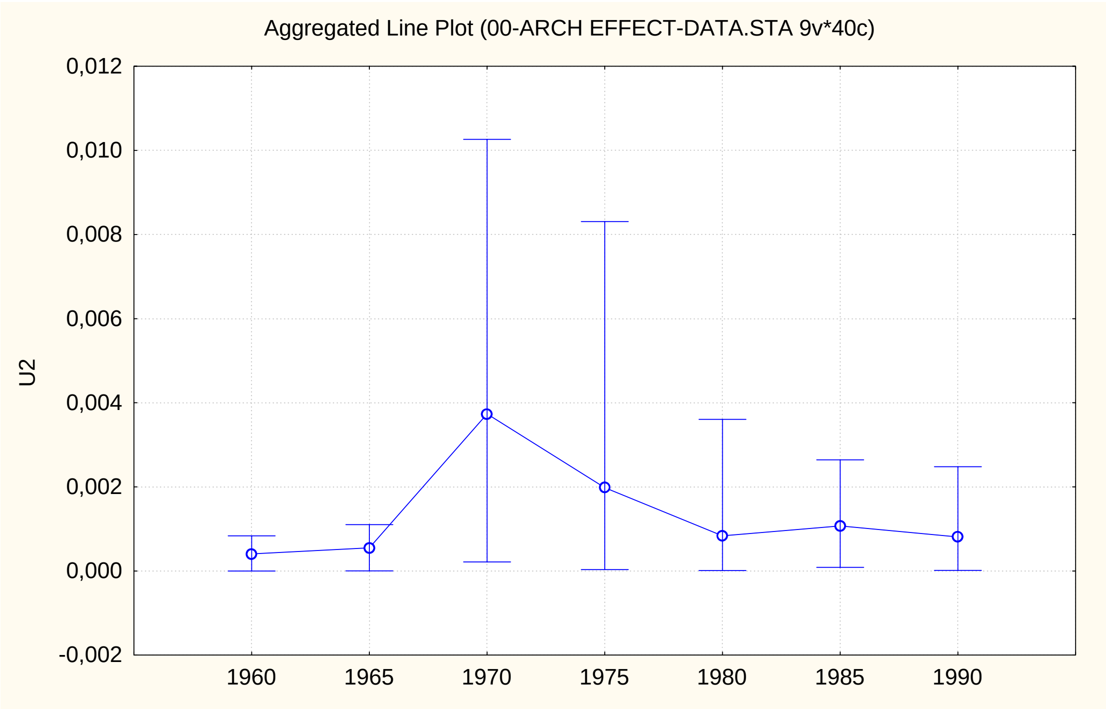
Regression Summary for Dependent Variable: RD						
	BETA	**ST. ERR.**	**B**	**ST. ERR.**	**T_31_**	**P_LEVEL**
Intercept			0,022623	0,012475062	1,813497671	0,079447128
RD_1	0,860284	0,091565	0,833502	0,088714894	9,39528964	0,00

The graphs of the squared regression residuals, the 5-year aggregated squared regression residuals and the regression residuals are shown in graphs 2, 3, 4 accordingly.

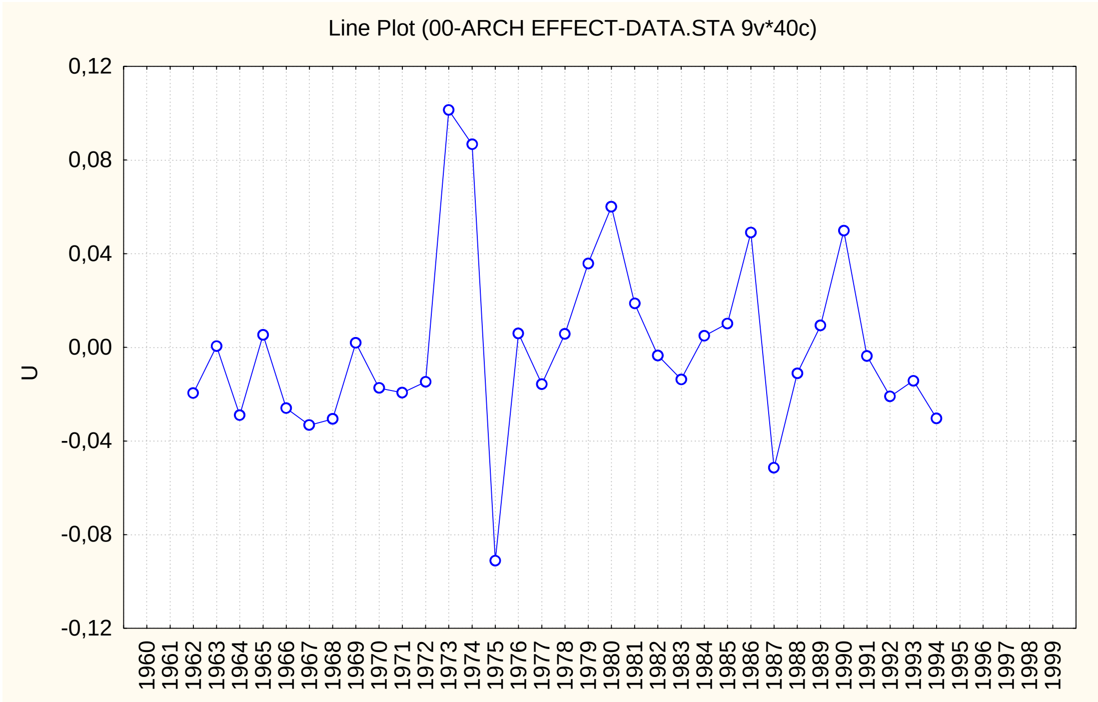
Graph 2: Squared regression residuals



Graph 3: Aggregated squared regression residuals



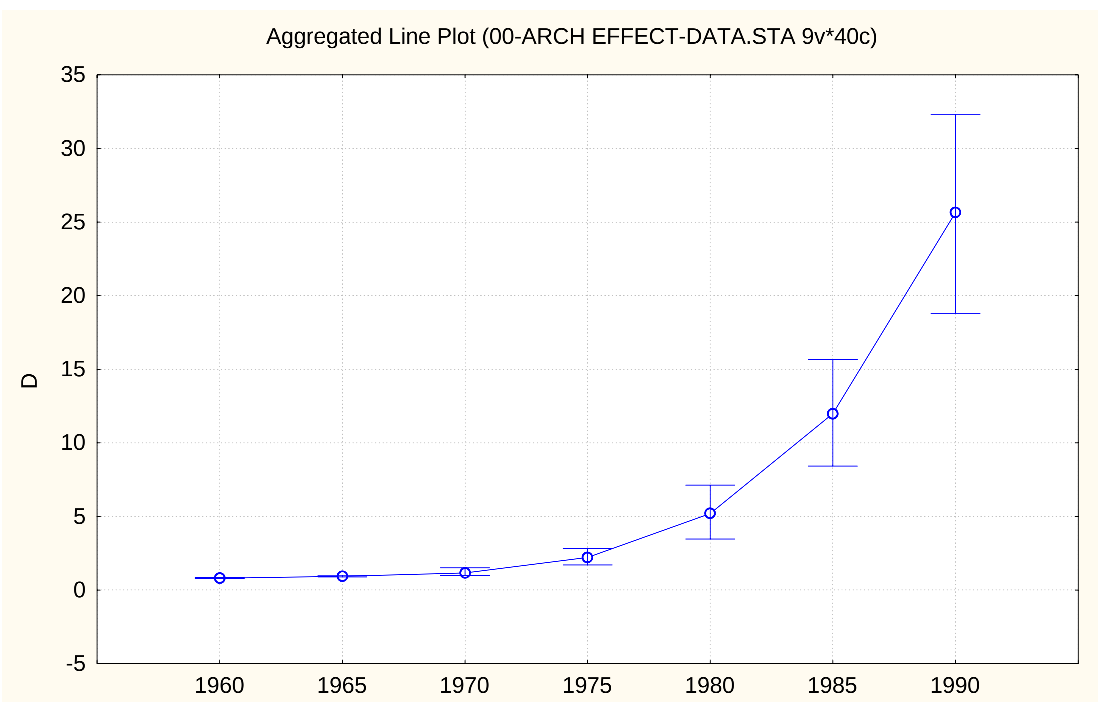
Graph 4: Regression residuals



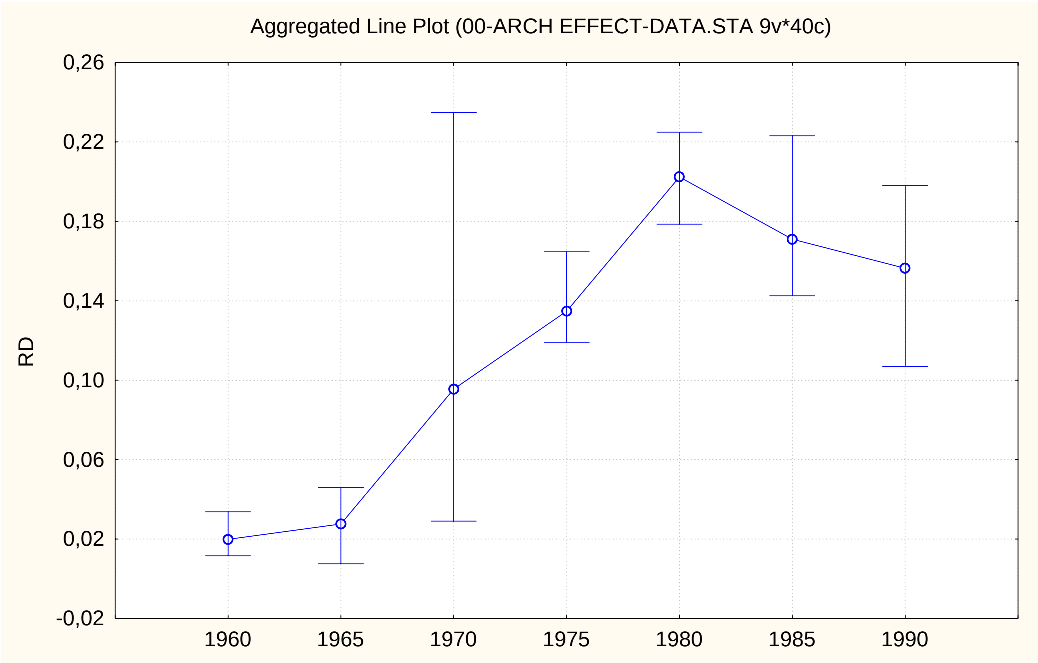
Application of the Arch Procedure and the Findings

The purpose of this part of the study is to detect ARCH effects in the variable RD. A look at figures 5 and especially 6, in which the data are aggregated in a 5-year basis, clearly shows that the variance changes.

Graph 5: Aggregated deflator

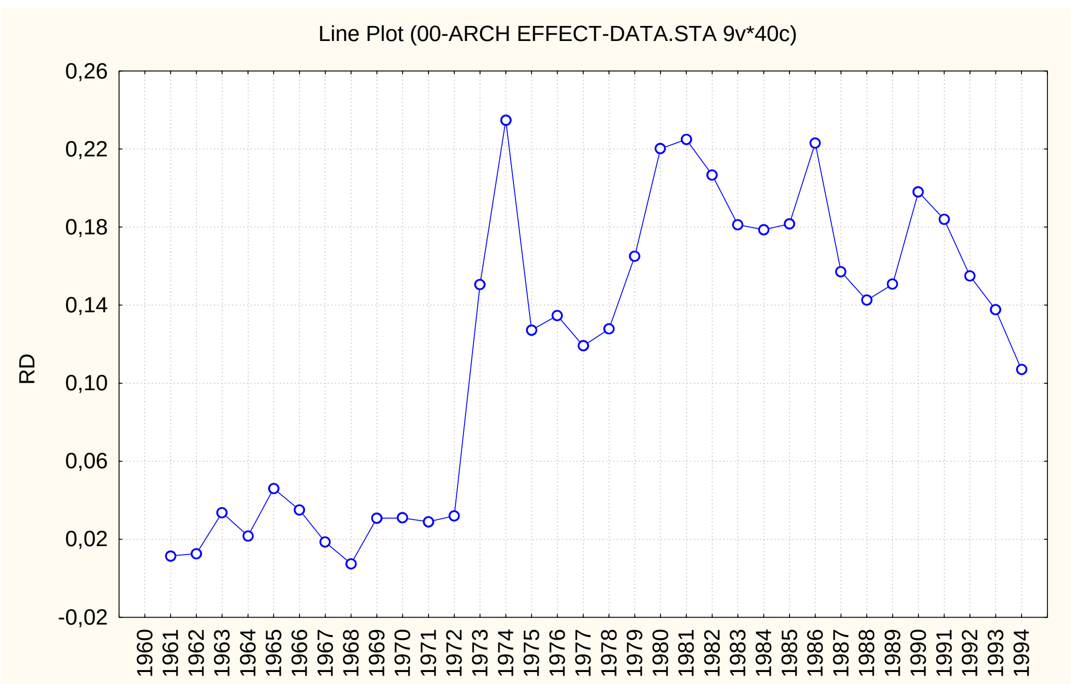


Graph 6: Aggregated RD



The following graph 7 shows the annually change rate in D (RD)

Graph 7: Change rate in D (RD)

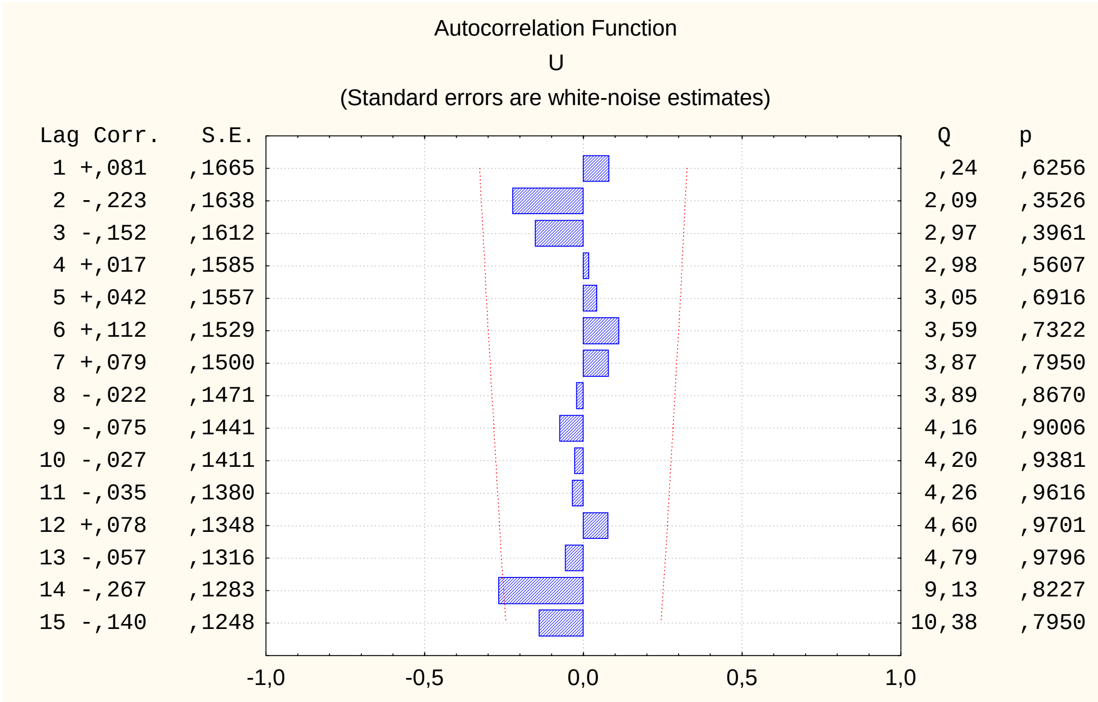


According to the stated theoretical analysis we first proceed to the regression in the model

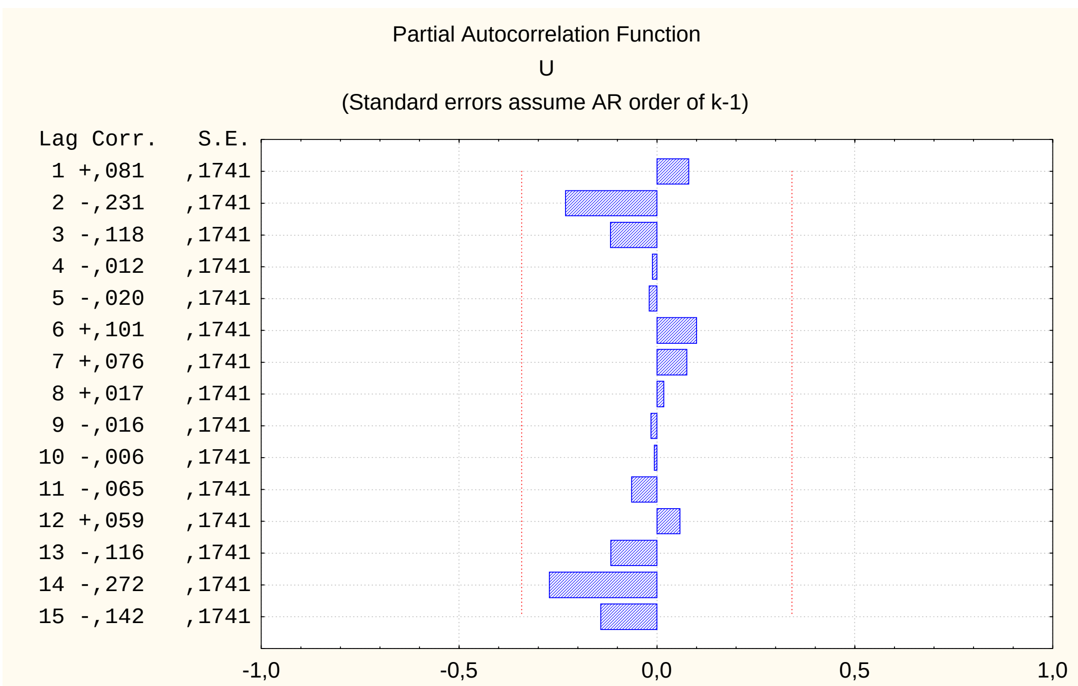
$$RD_t=\alpha_0+\alpha_1 RD_{t-1}+u_t \quad (10)$$

By this regression the regression residuals u_t are obtained. The figures 8 and 9, which exhibit the autocorrelation and the partial autocorrelation functions accordingly, clearly show that the residuals are not correlated.

Graph 8: ACF of regression residuals



Graph 9: PACF of regression residuals



Therefore, the condition of uncorrelated u's is fulfilled. We next check if the residuals exhibit an ARCH effect. For this purpose we apply the model (9) for p=1, 2, 3, 4 and we obtain 4 regression equations.

The test of the null hypothesis

H_0: $\alpha_0=\alpha_1=\ldots\ldots=0$ against the alternative hypothesis

H_1: not all α's are simultaneously zero

is performed by the Fisher's-F statistic at a 5% significance level

In the following tables 5, 6, 7 and 8 are shown the results for each regression from ARCH(1) to ARCH(4)

Table 5: Arch(1) regression results in arch(1) model

Regression Summary for Dependent Variable: U2						
	BETA	**ST. ERR.**	**B**	**ST. ERR.**	**T_30_**	**P_LEVEL**
Intercept			0,000789	0,00047	1,678437	0,103651
U2_1	0,454508	0,162627	0,453589	0,162298	2,794795	0,008966

Table 6: Arch(2) regression results in arch(2) model

Regression Summary for Dependent Variable: U2						
	BETA	**ST. ERR.**	**B**	**ST. ERR.**	**T_28_**	**P_LEVEL**
Intercept			0,000873	0,000516	1,692383	0,101674
U2_1	0,476452	0,188972	0,474291	0,188115	2,52129	0,017664
U2_2	-0,05702	0,188972	-0,05682	0,188308	-0,30173	0,765085

Table 7: Arch(3) regression results in arch(3) model

Regression Summary for Dependent Variable: U2						
	BETA	**ST. ERR.**	**B**	**ST. ERR.**	**T_26_**	**P_LEVEL**
Intercept			0,00117	0,000538	2,177281	0,038725
U2_1	0,454713	0,184165	0,454602	0,18412	2,469049	0,020436
U2_2	0,106907	0,203668	0,106732	0,203335	0,524908	0,604094
U2_3	-0,3446	0,184239	-0,34393	0,183881	-1,87041	0,072725

Table 8: Arch(4) regression results in arch(4) model

Regression Summary for Dependent Variable: U2						
	BETA	**ST. ERR.**	**B**	**ST. ERR.**	**T_24_**	**P_LEVEL**
Intercept			0,001148	0,000617	1,860927	0,075049
U2_1	0,482397	0,203155	0,480125	0,202198	2,374528	0,025912
U2_2	0,090443	0,211378	0,090306	0,211058	0,427875	0,672557
U2_3	-0,3832	0,211565	-0,38151	0,210636	-1,81125	0,082637
U2_4	0,074465	0,203055	0,07432	0,202661	0,366723	0,717039

Finally, for comparison reasons the results for all regressions are summarized in the following table 9

Table 9: Summary of regressions results

Regression model	**Regression results**							
	α_0	α_1	α_2	α_3	α_4	**AdjR^2**	**F**	**p-level**
$RD_t=\alpha_0+\alpha_1 RD_{t-1}$	0.022623	0.833502				0.7317	88.27	0.00000
ARCH(1)	0.000789	0.453589				0.1801	7.81	0.00087
ARCH(2)	0.000873	0.474300	-0.056800			0.1488	3.62	0.03984
ARCH(3)	0.001170	0.454602	0.106732	-0.343933		0.2174	3.68	0.02461
ARCH(4)	0.001148	0.480125	0.090306	-0.381515	0.074320	0.1886	2.62	0.05953

The p-value corresponding to the F-statistic is less than 0.05 (5%), which leads to the rejection of the null hypothesis for the parameters of the regression model (10) and the models ARCH(1), ARCH(2) and ARCH(3). Therefore, one cannot reject the hypothesis that the changing variance in the residuals follows an ARCH process.

Discussion of the Findings

A good scientific theory (and the applications of this theory) must always have in mind the conditions of its validity and, moreover, its application limits.

Therefore, the validity of our findings from the ARCH technique depends on the power of the technique, the proper application field (and, of course, on our proper use of it). The above consideration apply *par excellence* in econometrics, where the ignorance of what really happens in the investigated phenomenon leads the investigator to a despair theorization of the phenomenon loading it with assumptions, which are not at all self-explanatory or even plausible. In this study we have tried to detect ARCH effects in an economic time series. We did not reject existence of ARCH effects in our data. But did we really prove existence of such effects? Not at all - we simply did not reject their existence using standard techniques to test statistical hypotheses. But non rejection does not mean acceptance: not to reject the hypothesis that a yellow man be a Chinese does not imply that he is Chinese- he might very well be Japanese or even a yellow-painted Norwegian. In any case, statistics alone cannot prove or disprove any scientific theory. It is not its task. This is the task of the specialist of the phenomenon, which can be assisted by the statistics in measuring, correlating, forecasting. The ARCH models are complicated non-linear models, heavily loaded with assumptions concerning non-correlation of the variables involved, identical distributions etc. Are they the best in investigating heteroscedasticity? We believe that the subject is open but one *has* to use them, for these objections are common to the rest of the econometric techniques of the kind.

In the measure that we have correctly theorized the model and we have correctly read the regressions results *we cannot* reject existence of ARCH effects in the investigated time series, since we have no serious reasons to do so. From this moment on, we are in a position to proceed to point forecasts, to build confidence intervals and to measure the volatility of the series.

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